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Information Cascades and Threshold Implementation: Theory and an Application to Crowdfunding

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ABSTRACT

Economic interactions often involve sequential actions, observational learning, and contingent project implementation. We incorporate all-or-nothing thresholds in a canonical model of information cascades. Early supporters effectively delegate their decisions to a "gatekeeper," resulting in unidirectional cascades without herding on rejections. Project proposers can consequently charge higher prices. Proposal feasibility, project selection, and information aggregation all improve, even when agents can wait. Equilibrium outcomes depend on crowd size, and project implementation and information aggregation achieve efficiency in the large-crowd limit. Our key insights hold under thresholds in dollar amounts and alternative equilibrium selection, among other model extensions.

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FINANCING BUSINESS ACTIVITIES AND GATHERING SUPPORT often involve sequential contributions, observational learning, and project implementation contingent on achieving certain threshold levels of support. Crowd-based fundraising, which includes equity and reward crowdfunding, peer-to-peer lending, and initial coin offerings, constitutes the most salient recent example. Such economic interactions among sequential, privately informed agents are prone to information cascades that lead to incomplete information aggregation and suboptimal financing. Standard theories (e.g., Banerjee (1992), Bikhchandani, Hirshleifer, and Welch (1992)) focus on pure informational externalities with each agent's payoff structure independent of others' actions. We incorporate into a model of dynamic contribution games the fact that, in practice, many projects or proposals are implemented only with a sufficient level of support—an "all-or-nothing" (AoN) threshold. We show that threshold implementation drastically alters informational environments and economic outcomes, with implications for financing projects and aggregating information—arguably the two most important functions of modern financial markets.[1]

Specifically, we introduce threshold implementation in a standard framework of information cascade à la Bikhchandani, Hirshleifer, and Welch (1992). A project proposal is sequentially considered by N agents who choose to support or reject. Each supporter pays a prespecified contribution price and receives an eventual payoff normalized to one if the project is good. All agents are risk-neutral with a common prior belief about the project's quality. They each receive a private, informative signal and observe the actions of preceding agents before deciding whether to support. Deviating from the literature, we assume that supporters pay the price and receive the payoff if and only if the support level reaches an AoN threshold, which is either exogenously given or endogenously determined jointly with the price by the proposer.

AoN thresholds lead to unidirectional cascades in which agents never rationally ignore positive private signals to reject the project (i.e., there are virtually no DOWN cascades, which we define in the model), but may rationally ignore negative private signals to support the project (i.e., UP cascades are possible), making agents appear to have fears of missing out. Information aggregation also becomes more efficient, especially with a large crowd. With endogenous implementation threshold and price, the proposer no longer underprices the issuance, as seen in Welch (1992). Consequently, proposal feasibility (positive

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[1] An AoN threshold is common on crowdfunding platforms and in venture financing. Moreover, supermajority rule or q-rule is common in many voting procedures, assurance contract or crowdaction in public goods provision is characterized by sequential decisions and implementation thresholds (e.g., Bagnoli and Lipman (1989)), and charitable projects set target levels of fundraising to proceed (e.g., Andreoni (1998)).

probability for implementation), project selection (good projects are more likely implemented than bad projects), and information aggregation (public history of support revealing project quality) all improve. In particular, when the number of agents approaches infinity, equilibrium project implementation and information aggregation become efficient, which is in stark contrast to findings in prior literature on information cascades (Banerjee (1992), Lee (1993), Bikhchandani, Hirshleifer, and Welch (1998), Ali and Kartik (2012)).

To derive these results, we first take the AoN threshold and price as given in the subgame of agent contribution and learning. We show that before reaching the threshold, the aggregation of private information stops only upon an UP cascade. The intuition is that the AoN threshold links agents' payoffs to their subsequent actions, making them internalize part of the informational externalities of their actions. Such forward-looking considerations lead to interesting asymmetries: Even before an UP cascade, agents with positive private signals always support because they essentially delegate decisions to a future "gatekeeping" agent whose support decision brings the total support to the threshold. This delegation hedges against mistakenly supporting a bad project because the subsequent gatekeeper makes a more informed contribution decision by observing a longer sequence of previous actions. DOWN cascades are therefore always interrupted by agents as positive signals before the AoN threshold is reached. In contrast, an agent with a negative signal is reluctant to support a project before it reaches the AoN threshold or an UP cascade, for fear that supporting the project (which would now be indistinguishable from the actions of agents with positive signals) may mislead subsequent agents to positively update their beliefs about the project's quality in spite of the negative signal the agent privately observes. This agent's supporting action would then increase the likelihood of a bad project being funded, reducing her expected payoff. However, when the agent's belief about the project's quality is sufficiently high, an UP cascade starts and the agent no longer worries about misleading subsequent agents because they do not positively update based on her action anyway.

We next analyze how the entrepreneur or proposer endogenously designs the AoN threshold and the contribution price to maximize the level of support. A higher AoN threshold, albeit less likely to be reached, delays potential DOWN cascades because as we argue earlier, a DOWN cascade cannot happen before the AoN threshold has been reached. The entrepreneur's optimal AoN is thus set to be just sufficient that achieving it implies a high valuation relative to the contribution price and essentially excludes DOWN cascades. Meanwhile, the proposer trades off increasing the proceeds from supporters (by charging a higher price) with lowering the AoN (and charging a correspondingly lower price so as to still effectively exclude DOWN cascades) to boost the probability of implementing the project. In general, a larger crowd mitigates the concern about implementation failure and generally permits a higher optimal price, making prices depend endogenously on crowd size.

AoN thresholds and unidirectional cascades have three important implications. First, they improve project feasibility by allowing good projects with high production costs to be supported. Standard information cascade theories suggest that for projects with high production costs, the contribution price necessary to at least cover the cost is so high that the first agent will reject it even with a positive private signal, resulting in a DOWN cascade and guaranteed funding failure (Welch (1992)). AoN thresholds mitigate concerns about DOWN cascades, making it possible to charge a high price to cover the production costs. Second, AoN thresholds improve project implementation efficiency because charging a high price implies implementation only when the posterior belief is sufficiently positive, which is correlated with the project's positive quality. Third, AoN thresholds facilitate information aggregation by mitigating DOWN cascades and delaying the arrival of UP cascades. A proposer facing a large number of potential supporters can use threshold implementation to guard against DOWN cascades and to charge a high contribution price (which delays UP cascades) for greater proceeds or support, regardless of whether the threshold is eventually reached.

While outcomes in standard models of information cascades are independent of the size of the agent base, the case with AoN thresholds is different: missupporting or misrejecting errors decrease with crowd size, and the endogenous price converges to the highest level at which the proposer extracts full surplus. In the limit, projects are implemented if and only if they are of high quality. Public knowledge about the project's true type also becomes perfect. We therefore obtain socially efficient project implementation (under private signals) and full information aggregation with a large crowd, hitherto unachievable in most models of information cascades. These findings are especially relevant in the age of digital platforms and the Internet, which feature outreach to extremely large crowds.

We demonstrate that our key insights apply even when agents have the option to postpone their decisions or to delay expending effort to acquire information, and thus are less subject to the usual critiques of exogenous action timing. We also show that our findings are robust to introducing investor heterogeneity and thresholds based on dollar amounts (and to introducing small contribution frictions or learning costs, as discussed in the Appendix). We further analyze other perfect Bayesian Nash equilibria (PBNE) under the same mild tie-breaking convention to understand the strategic complementarity introduced by AoN thresholds. In terms of project implementation and information aggregation, the equilibrium outcomes converge to those characterized in our baseline model.

The theoretical insights that we derive apply to many sequential contribution games such as venture financing or syndicated loans. We highlight the application to crowdfunding for several reasons. First, crowdfunding has quickly become a mainstream source of capital for entrepreneurs, with its total volume surpassing the market size for angel funds in 2015 and reaching a whopping 35 billion USD globally in 2017 even before the explosion of crypto-token offerings. Second, it presents a setting in which the technology allows for outreach to large crowds, which renders the limiting results for large crowds relevant and important. Third, the sequential nature of contributions and threshold implementation are salient in crowdfunding, making it representative of general dynamic economic interactions with observational learning and threshold implementation, unlike auctions.

Other forms of entrepreneurial or corporate finance also feature investors frequently inquiring about preceding investments as well as threshold implementation written as clauses in contingency offering contracts, initial public offerings (IPOs), subscription money-back guarantees, or private placement memoranda.[2] These settings can also be analyzed through our conceptual lens, further demonstrating the practical importance of threshold implementation design in a variety of economic interactions and financing situations.

Literature. Our paper adds to the theory of informational cascades, sequential decisions, and observational learning. The insights from prior dynamic informational models primarily concern signal structure and learning bias (Banerjee (1992), Bikhchandani, Hirshleifer, and Welch (1992), Welch (1992), Bikhchandani, Hirshleifer, and Welch (1998), Chamley (2004), Callander (2007), Aghamolla and Hashimoto (2020)). Traditionally, informational cascades can be asymmetric or even unidirectional only when some actions are not observable (Chari and Kehoe (2004), Guarino, Harmgart, and Huck (2011), Herrera and Hörner (2013)). Our contributions to this literature are twofold. First, we obtain asymmetric informational cascades endogenously due to threshold implementation even with observable actions. Second, we show that full learning can be achieved with bounded signals once we allow for payoff interdependence via threshold implementation. Importantly, we obtain perfect information aggregation in large-crowd limits, which is typically unachievable with information cascades (Ali and Kartik (2012)). Our model therefore describes a new set of equilibrium behavior by large crowds and adds to our understanding of how the latest technologies, such as the Internet and blockchain, impact social efficiency in information aggregation and fundraising in financial markets.

Our paper also adds to an emerging literature on AoN design in the context of crowdfunding and marketplace lending. Strausz (2017) and Ellman and Hurkens (2019) find that AoN is crucial for mitigating moral hazard and enhances price discrimination. Chemla and Tinn (2020) share Strausz's (2017) concerns about moral hazard, but also emphasize the real option of learning through crowdfunding. Chang (2020) shows that in simultaneous move games as in Chemla and Tinn (2020), AoN also generates more profit under common-value assumptions by making the expected payments positively

[2] In an angel or A round of financing, investors who are approached later in the fundraising process often learn which other financiers indicated support for the project and offer additional contributions on the condition that the fundraising reaches certain thresholds (Halac, Kremer, and Winter (2020)). In an IPO, late investors learn from the behavior of early investors, and the issuer may choose to withdraw the offering if the market reaction is lukewarm (e.g., Ritter and Welch (2002)). Indeed, in the early 1980s, many tiny firms in the United States conducted an IPO with a best-efforts contract that frequently had an AoN feature. We thank Jay Ritter for providing this example and Steve Kaplan for pointing us to sample proprietary documents of private placement memoranda.

correlated with values. Hakenes and Schlegel (2014) argue that endogenous loan rates and AoN thresholds encourage information acquisition by individual households in lending-based crowdfunding. Brown and Davies (2020) focus on a simultaneous-action setting in which threshold implementation, when set by an entrepreneur after observing the total contribution, creates a losers' blessing that discourages investors from acquiring information and reduces financing efficiency.

Instead of introducing moral hazard or financial constraints, we offer the first dynamic model of sequential contribution under threshold implementation. Our emphasis on observational learning, a salient feature of crowdfunding and support-gathering processes in real life, distinguishes our paper from and complements existing crowdfunding literature such as Kremer (2002) and García and Urošević (2013).[3] While demand is exogenous in Strausz (2017), in our setting demand during the crowdfunding campaign is endogenously determined by both the true underlying quality of the project and dynamic learning under informational frictions. We also confirm the superiority of AoN designs over "keep-it-all" designs in a dynamic environment and the value of committing to threshold implementation for improving financing efficiency (for which Brown and Davies (2020) provide an example under simultaneous actions) and information aggregation.

Models of dynamic learning become complicated very quickly. With respect to the particular application of our theory, we do not claim to cover all aspects of crowdfunding, especially those concerning information acquisition and information design (e.g., Kremer, Mansour, and Perry (2014); Glazer, Kremer, and Perry (2015)). Our paper should be viewed as a first step in understanding the consequences of introducing threshold implementations in dynamic contribution games with large crowds. Instead of allowing the entrepreneur to possess private information about production costs as in Strausz (2017), we emphasize the aggregation of investors' private signals about project quality. Whereas Brown and Davies (2020) emphasize investors' information acquisition, we focus on entrepreneurs' ex ante commitment to implementation thresholds in affecting information aggregation and we derive the optimal thresholds in a dynamic setting.

The rest of the paper is organized as follows. Section I sets up the model. Section II characterizes the equilibrium, starting with the subgame of contribution to illustrate the main mechanism before endogenizing contribution prices and implementation thresholds. Section III discusses model implications on proposal feasibility, project selection, and information aggregation. Section IV extends the model to allow options to wait, budget heterogeneity and thresholds in dollar amounts, and characterizations of other equilibria.

[3] An average crowdfunding campaign lasts nine weeks or longer (https://blog.fundly.com/crowdfunding-statistics/). As Canal (2020) notes, one of the advantages of crowdfunding platforms is that "users can see the success of a campaign as it progresses," not to mention the ample empirical evidence they offer on agents' sequential arrivals (e.g., Vismara (2018), Deb, Oery, and Williams (2022)).

Section V concludes. The Appendix contains proofs. Details on various model extensions are available in the Internet Appendix.[4]

I. A Dynamic Model of Crowd-Based Support Gathering

A. Model Setup

Consider a project proposal presented to agents $i = 1, 2, \ldots, N$ who sequentially take actions $a_i \in \{-1, 1\}$ to either support ($a_i = 1$) or reject ($a_i = -1$) it.[5] In the crowdfunding setting, supporting means contributing financially; more broadly, supporting can be interpreted as adopting or advocating for certain behaviors by incurring a personal cost. If the proposal is implemented, then the proposer collects from every supporting agent a prespecified "contribution" p, and in exchange each agent ultimately receives a project payoff of V, which is either zero or one.[6] Given that crowdfunding often serves a demand discovery function (Strausz (2017)), V can be interpreted as a crude transformation of the uncertain aggregate market demand, which could be high ($V = 1$) or low ($V = 0$).

A.1. Threshold Implementation

We depart from prior literature on information cascades by incorporating AoN thresholds commonly observed in practice. The proposer receives "all" contributions if the campaign reaches a prespecified threshold level of support and "nothing" otherwise.[7] Put differently, the project is implemented if and only if at least T agents support it, where the threshold T could be exogenous, for example, driven by the need to cover a minimum project scale that is outside the entrepreneur's control, once the contribution price is specified. In

[4] The Internet Appendix is available in the online version of this article on *The Journal of Finance* website.

[5] We use the terms "support" and "invest" interchangeably, although our model can be applied to any situation in which the contribution is nonpecuniary. In practice, crowdfunders typically observe both the total capital raised and the number of supporters to date (Vismara (2018)), but this distinction is immaterial in the baseline model. Importantly, our setting differs from that for voting because noncontributors do not bear any project risk whereas nonvoters typically do face the consequences of a voting outcome.

[6] A separate literature allows price to change dynamically and focuses on asset pricing implications (Avery and Zemsky (1998); Brunnermeier (2001); Vives (2010); Park and Sabourian (2011)). We follow the standard cascade models to fix the price for taking an action ex ante, which closely matches applications in crowdfunding and entrepreneurial finance. In other activities such as political petitions, p can be interpreted as the supporting effort or reputation cost if the petition goes through and becomes public.

[7] The JOBS Act mandates that crowdfunding platforms adopt threshold implementation (Sec. 4A.a.7. See http:// beta.congress .gov / bill / 112th- congress / senate- bill / 2190 / text). The AoN mechanism, alternatively known as a "provision point mechanism," has also been used in Regulation D filings since 1982 (Bagnoli and Lipman (1989)). As in Hakenes and Schlegel (2014) and Chang (2020), we assume that an entrepreneur can commit ex ante to an implementation threshold.

many cases (including crowdfunding), however, T is endogenously set by the entrepreneur, which is equivalent to setting a total dollar amount when agents face the same contribution price. We discuss thresholds in dollar amount terms when we consider investor heterogeneity in Section IV.B. Note that supporters pay p only when the project is implemented. Threshold implementations are a salient feature of crowdfunding markets, and our contribution is to provide insights on their informational effects, especially concerning financing and information aggregation outcomes.

A.2. Agents' Information and Decision

All agents (indexed by i) and the proposer are rational, risk-neutral, and share the common prior that the project pays $V = 0$ and $V = 1$ with equal probability. Our specification well describes equity-based crowdfunding and peer-to-peer lending, which constitute 80% of the entire crowdfunding market as of 2020. Even in the case of reward-based crowdfunding, whereby agents have private valuations and idiosyncratic preferences, there is a common value that corresponds to the basic quality of the product. While our specification does not fully capture cases such as sales of art or music for which private value dominates, the common-value assumption allows unambiguous comparisons concerning project implementation and information aggregation with prior studies (e.g., Wit (1997)).

Each agent i observes one conditionally independent informative private signal $x_i \in \{1, -1\}$ such that

$$\Pr(x_i = 1|V = 1) = \Pr(x_i = -1|V = 0) = q \in \left(\frac{1}{2}, 1\right). \tag{1}$$

We denote the sequence of private signals by $x = (x_1, \ldots, x_N)$ and the set of all such sequences by $X = \{1, -1\}^N$.[8]

The order of agents' decision making is exogenous and known to all.[9] When agent i makes her decision, she observes x_i and the history of actions $\mathcal{H}_{i-1} \equiv (a_1, a_2 \ldots, a_{i-1}) \in \{-1, 1\}^{i-1}$. Her strategy can thus be represented as $a_i(\cdot, \cdot) : \{1, -1\} \times \{-1, 1\}^{i-1} \to \Delta(\{-1, 1\})$, which includes mixed strategies in terms of probability distributions of the action set $\{-1, 1\}$. To simplify

[8] The binary information and action structure here are standard in the literature (Bikhchandani, Hirshleifer, and Welch (1992)). In Section III of the Internet Appendix, we show that the main results and intuition are robust to considering multiple investment amounts and asymmetrically distributed signals.

[9] While real-world examples such as crowdfunding may involve endogenous orderings of agents, our setup allows for a comparison with the large literature on information cascades, which typically assumes exogenous orders of agents (Kremer, Mansour, and Perry (2014)). Moreover, because, in practice, agents update their beliefs based on the passage of campaign time (see Herrera and Hörner (2013)) and use contribution information alone to predict final funding outcomes (Dasgupta et al. (2020)), our setup can capture the case in which agents can roughly determine their position in line by referencing the usual accumulation and rejection with the passage of calendar time. In Section IV.A, we show that our key findings are robust to agents having the option to wait.

exposition, we define $A_i = \sum_{j=1}^{i} a_j \mathbb{1}_{\{a_j=1\}}$, for $1 \leq i \leq N$, as the total number of supporters up to agent i. When $1 \leq i' < i \leq N$ and $\mathcal{H}_{i'}$ has the same first i' elements as $\mathcal{H}_i$, we say that $\mathcal{H}_i \in \{-1, 1\}^i$ *nests* $\mathcal{H}_{i'} \in \{-1, 1\}^{i'}$, a concept that we use in an equilibrium definition below. Agent i's optimization is given by

$$\max_{a_i \in \{-1,1\}} \mathbb{1}_{\{a_i=1\}} \mathbb{E}\big[(V - p)\mathbb{1}_{\{A_N \geq T\}} \mid x_i, \mathcal{H}_{i-1}, a_i = 1\big], \tag{2}$$

where A_N is the total number of supporters among all agents, and $\mathbb{1}_{\{A_N \geq T\}}$ is the indicator function for project implementation. Agent i gets zero payoff from rejecting the proposal ($a_i = -1$) and $(V - p)\mathbb{1}_{\{A_N \geq T\}}$ from supporting the proposal ($a_i = 1$).[10] The parameter $a_i = 1$ appears in the conditioning term in (2) because, given equilibrium strategies of other agents a^*_{-i}, subsequent agents' decisions and project implementation generally depend on agent i's action.

Following the convention in the literature (e.g., Banerjee (1992); Bose, Orosel, Ottaviani, and Vesterlund (2008)), we introduce a tie-breaking rule.

ASSUMPTION 1 (Tie-breaking): *When the AoN threshold can still be reached if all remaining agents support, an agent indifferent between supporting and rejecting always supports.*

Threshold implementations introduce strategic complementarity of agents' actions, which naturally creates equilibrium multiplicity. For example, one trivial equilibrium entails everyone rejecting and the project failing for sure regardless of agents' private signals. So threshold implementations do introduce a downside. Assumption 1, which is an equilibrium refinement, rules out such equilibria that are undesirable in terms of implementation outcomes.[11]

Moreover, if one cares about information aggregation even when the project is not implemented, one faces redundant equilibria that share the same implementation outcomes and payoffs, but with different public information sets when AoN cannot be reached. We therefore need the following weak refinement.

ASSUMPTION 2: *In equilibria in which the crowdfunding fails (the AoN threshold is not reached), an agent supports if and only if she has a positive private signal.*

This assumption allows information aggregation to be strictly monotone in A_N (e.g., Proposition 6) and to achieve social efficiency in the large-crowd limit, even when the crowdfunding fails. It is not needed for other results.

[10] We implicitly assume free information acquisition and no contribution cost in the baseline model. We extend our model to (small) contribution/information acquisition costs in Section II of the Internet Appendix.

[11] We discuss such undesirable equilibria when we extend our model to allow for contribution or information acquisition costs in Section II of the Internet Appendix.

A.3. Proposer's Optimization

Let $\nu \geq 0$ be the cost per supporter incurred by the proposer. It can be thought of as the production cost of each product with reward-based crowdfunding or the private valuation (outside option) of an issuer's shares when the project is funded without equity-based crowdfunding. We assume $\nu < \frac{q^N}{q^N+(1-q)^N}$, that is, the cost is below the expected investment payoff when it is public that all agents have positive signals; otherwise, the project trivially fails to be implemented. Given the campaign length N (the proposer cannot terminate prematurely), a proposer chooses price p and AoN threshold T to solve

$$\max_{p,T} \pi(p, T, N) = \mathbb{E}\big[(p-\nu)A_N \mathbb{1}_{\{A_N \geq T\}}\big]. \tag{3}$$

The proposer's expectation depends on investor agents' equilibrium strategies $\{a_i^*\}_{i=1,2...,N}$. In the case of fundraising, the proposer maximizes his expected profit; in nonfinancial scenarios, the proposer solicits the maximum amount of support, with p interpreted as each agent's additive amount of support. We leave alternative funding objectives to future studies.

When an agent's action does not reflect her private signal, the market fails to aggregate her dispersed information. Our notion of informational cascade is then rather conventional (e.g., Bikhchandani, Hirshleifer, and Welch (1992)).

DEFINITION 1 (Information Cascade): An *UP* cascade occurs following a history of actions $\mathcal{H}_n$ ($1 \leq n < N$) if along the equilibrium path, all subsequent agents support the proposal, regardless of their private signal, while Agent n herself is not part of any cascade. We denote the set of such histories by $\mathbb{H}^U$. A *DOWN* cascade is similarly defined, replacing "support" with "reject," and $\mathbb{H}^U$ with $\mathbb{H}^D$.

Standard models feature both UP and DOWN cascades. If a few early agents observe positive signals, their support may push the posterior so high that the project remains attractive even for someone with a private negative signal. Similarly, a series of negative signals may doom the offering. An early preponderance toward supporting or rejecting the projects leads all subsequent individuals to ignore their private signals, which are then never reflected in the public pool of knowledge.

B. Equilibrium

We use the concept of PBNE. Each agent's equilibrium action strategy $a_i^*(x_i, \mathcal{H}_{i-1}, p^*, T^*)$ is a function of her own private signal x_i, the history $\mathcal{H}_{i-1}$, and the proposer's choice $\{p^*, T^*\}$. Strategic complementarity among agents introduces multiple equilibria that survive Assumption 1. In our baseline analysis, we focus on a subset of equilibria in which all actions are informative outside a cascade, as we formalize next.

DEFINITION 2 (Informer Equilibrium): An equilibrium is an "informer equilibrium" if for every i and history $\mathcal{H}_{i-1} \in \{-1, 1\}^{i-1}$ that does not nest any history in $\mathbb{H}^U$ or $\mathbb{H}^D$, agent i's action varies with x_i, that is, $a_i(1, \mathcal{H}_{i-1}) \neq a_i(-1, \mathcal{H}_{i-1})$.

In other words, informers' actions are informative before an information cascade. Informer equilibria are intuitive and clearly illustrate our economic insights. In Section IV.C, we analyze all other PBNEs that survive Assumption 1 and establish that these variants asymptotically converge ($N \to \infty$) to the "informer equilibrium" in pricing, project implementation, and information aggregation.

C. Benchmark without Threshold Implementation

Consider a benchmark without threshold implementation (equivalently, $T = 1$), as in Bikhchandani, Hirshleifer, and Welch (1992) and Welch (1992).[12] The project is implemented for sure and an agent's payoff does not depend on subsequent agents' actions. Agent i chooses to support if and only if $\mathbb{E}[V|x_i, \mathcal{H}_{i-1}] \geq p$.

With exogenous p, both UP and DOWN cascades can occur, which halt information aggregation. With endogenous p, imprecise signals can cause "underpricing."

LEMMA 1: *The proposer always charges $p \leq q$. In particular, when $\nu = 0$ and $q \leq \frac{3}{4} + \frac{1}{4}(3^{\frac{1}{3}} - 3^{\frac{2}{3}}) \approx 0.59$, the optimal price is $p^* = 1 - q < \frac{1}{2} = \mathbb{E}[V]$.*

Lemma 1 generalizes the underpricing results ($\nu = 0$) in Welch (1992). The pricing upper bound q is not tight but rather reflects the possibility of early DOWN cascades.[13] If $p > q$, then even with a positive signal $x_1 = 1$, the first agent rejects and so does every subsequent agent, yielding zero payoff for the proposer. The second part of the lemma concerns optimal pricing when agents' signals are imprecise. UP and DOWN cascades affect the proposer's payoff asymmetrically because he benefits from UP cascades by attracting support from future agents with negative signals while DOWN cascades mean that a few early rejections may doom his offering. Consequently, he optimally underprices, $p = 1 - q < \frac{1}{2}$, to ensure an UP cascade from the first agent, even in the presence of a negative signal. Note that the proposer's incentive to create an UP cascade dominates because the signal is imprecise, that is, he does not have to lower the price much to trigger an UP cascade. Moreover, at the optimal price $p = 1 - q$, an UP cascade is completely uninformative about project value.

[12] Welch (1992) acknowledges that internalization channels such as threshold implementation could affect the results and expounds on the possibility that a project's net present value is lost unless sufficiently many investors subscribe. We formalize and enrich the discussion with an emphasis on the types of cascades that would occur in equilibrium.

[13] Without a bound on q, the proposer's optimal price is either $1 - q$ or $\frac{1}{2}$.

II. Equilibrium Characterization

We now solve the equilibrium in several steps. First, we examine agents' decisions taking the price p and AoN threshold T as given. We then derive the proposer's endogenous p and T prior to the crowdfunding and compare the equilibrium outcomes to the benchmark outcomes without implementation thresholds.

A. An Illustration with $T = 2$

Before providing a formal analysis, we illustrate the key economic intuition through a simple example. Consider a funding campaign with $p \in (\frac{1}{2}, q)$ and $N = 3$. As in Bikhchandani, Hirshleifer, and Welch (1992), without any AoN threshold, an UP cascade occurs when both agents 1 and 2 observe positive signals and choose to support, and a DOWN cascade starts when agent 1 observes a negative signal and chooses to reject, which leads to agents 2 and 3 to follow suit regardless of their own signals.

Now suppose that we set the AoN threshold to $T = 2$. When both agents 1 and 2 receive positive signals and support, agent 3 continues to find the project attractive regardless of her private signal, that is, threshold implementation does not affect the UP cascade. When agent 1 observes a negative signal and rejects the proposal, however, a DOWN cascade no longer occurs. If agent 2 observes a positive signal, her optimal choice is to support because she pays the price p only when agent 3 chooses to support as well. If agent 3's signal is negative, then she finds the project unattractive and chooses to reject, and the proposal fails to be implemented. Anticipating agent 3's action, agent 2 effectively pays p only when agent 3 observes a positive signal as well, which implies that the project is profitable. In a sense, agent 2 is hedged from the downside risk of supporting.

Note that this argument does not imply that agent 2 supports regardless of her private signal, because agent 2's decision is informative for agent 3, and agent 2 does not want to mislead agent 3. To see this, say that agent 1 supports. Agent 2 would have incentive to support if she observes a negative signal because deviating to support would lead agent 3 to (incorrectly) infer that agent 2's signal is positive and (incorrectly) choose to support and implement the proposal even if she observes a negative signal.

When the AoN threshold is about to be reached (i.e., when there exist $T - 1$ supporting agents), we are back to a standard information cascade setting and both UP and DOWN cascades could happen. To see this, suppose $N = 4$ while $T = 2$. If agent 1 rejects and agent 2 supports, and agent 3 observes a negative signal and chooses to reject, then there is a DOWN cascade and agent 4 will reject regardless of her private signal. However, given agent 2's support, the project is certainly implemented if agent 4 contributes.

B. Observational Learning under Threshold Implementation

As seen in the simple example, no DOWN cascade occurs before the AoN threshold is approached. When the AoN threshold is about to be reached, the model reduces to a standard cascade setting and both UP and DOWN cascades become possible. Define k_i as the difference between the number of inferred positive signals and inferred negative signals up to agent i. Then, k_i depends on both the history $\mathcal{H}_i$ and agents' equilibrium strategies (which affect how informative the action history is). Similar to Bikhchandani, Hirshleifer, and Welch (1992), below we show that the public posterior valuation of the project follows

$$\Pr(V = 1|\mathcal{H}_i) = \frac{q^{k_i}}{q^{k_i} + (1-q)^{k_i}} \equiv V_{k_i}. \tag{4}$$

We define $\bar{k}(p) \equiv \min\{k \mid p \leq V_k, k \in \mathbb{Z}\}$ be the minimum number of "excess" inferred positive signals needed to form a posterior valuation that is weakly higher than the contribution p, where $\mathbb{Z}$ denotes the set of integers. We then have the following proposition.

PROPOSITION 1: *For any given pair of (p, T), a unique informer equilibrium exists. In equilibrium, when there is no cascade yet, an agent supports if and only if the private signal is positive. Importantly, an UP cascade occurs following history $\mathcal{H}_i$ when $k_i \geq \bar{k}(p) + 1$, while a DOWN cascade occurs following history $\mathcal{H}_i$ only when $A_i \geq T - 1$ and $k_i < \bar{k}(p) - 1$.*

As in the earlier example, Proposition 1 shows that there is no DOWN cascade before approaching the AoN threshold ($A_{i-1} < T - 1$) and all actions are informative unless a cascade immediately follows. When $A_{i-1} = T - 1$, agent i and subsequent agents face exactly the same decision as in standard cascade models, and they update beliefs accordingly.

In stark contrast to the benchmark without threshold implementation, DOWN cascades disappear before approaching the AoN threshold ($A_{i-1} < T - 1$) because an agent with a positive signal is protected when supporting, in that, she does not pay if the project turns out to be bad and does not achieve enough support to be implemented. The agent observing $T - 1$ prior supporters is the "gatekeeper" for all prior supporters because her decision affects whether other supporters incur the contribution cost and receive the project payoffs. Because she observes a longer history and makes a more informed decision, prior supporters benefit from "delegating" the implementation decision to her.

Having said that, observing a longer history is helpful only when actions convey private information. To complete the argument above, we argue that when there is no UP cascade yet and before the AoN threshold is approached, agents with negative signals reject the proposal: If an agent with a negative signal deviates and supports, then all subsequent agents would misinterpret her action and form incorrect posterior beliefs. The overoptimistic beliefs would imply that subsequent agents start an UP cascade too early or reach the AoN threshold when the true posterior is not high enough. As a result, agents with negative signals find deviations unattractive. Interestingly, here the

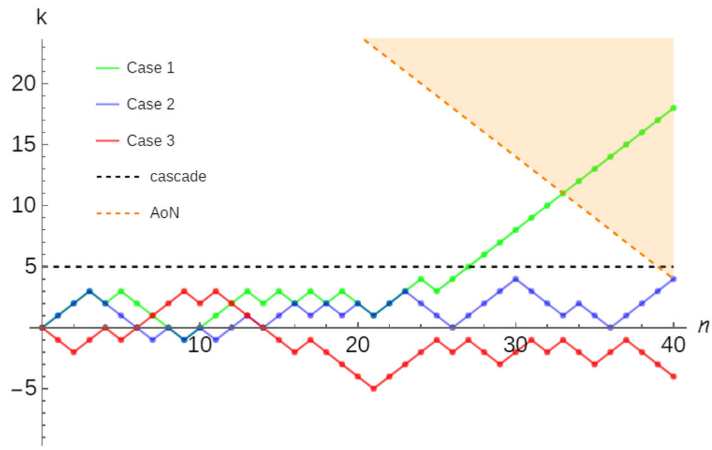


Figure 1. Evolution of support-reject differential. This figure plots simulated paths for $N = 40$, $q = 0.7$, $p^* = V_4 = 0.9673$, and AoN threshold $T^*(N) = 22$. Case 1 indicates a path that crosses the cascade trigger $\bar{k}(p) + 1 = 5$ at the 26^{th} agent, with all subsequent agents supporting regardless of their private signal. Case 2 indicates a path with no cascade, but the project is still funded by the end of the fundraising. Case 3 indicates a path where the AoN threshold is not reached and the project is not funded. The orange shaded region above the AoN line indicates that the project is funded. (Color figure can be viewed at wileyonlinelibrary.com)

information breakdown associated with UP cascades facilitates information aggregation before $\bar{k} + 1$ is reached.

Note that an UP cascade can still occur before reaching the AoN threshold due to the asymmetry between the payoffs of supporting versus rejecting. A rejecting agent does not share the upside if the eventual posterior valuation makes supporting profitable and the project is implemented. Yet, a supporting agent is hedged from potential losses, making agents more motivated to support (and potentially creating UP cascades). We illustrate the two scenarios (with and without an UP cascade) with project implementation in Figure 1, which plots the difference between supporting agents and rejecting agents when n agents have arrived. The figure also includes a sample path of eventual implementation failure (i.e., the AoN threshold is not reached).

B.1. Exogenous Price or AoN Threshold

Proposition 1 characterizes the subgame-perfect equilibrium from which we build to endogenize p and T. It also provides insights into the situation in which the entrepreneur has little say in the price and funding target. For example, T could be the minimal capital required to fund a film production that is exogenous to the model. Because the proposition holds for any p and T, the asymmetry in information cascades holds whenever we have threshold implementation.

C. Optimal Price and AoN Threshold

In practice, project proposers and entrepreneurs typically set contribution prices and implementation thresholds. We next endogenize the proposer's choices in the model. We find two main effects. First, they essentially rule out DOWN cascades even after the thresholds are approached. Second, they make the optimal price depend on the number of agents. The two effects jointly affect proposal feasibility, project selection, and information aggregation, especially with large crowds.

PROPOSITION 2: *For each N, an informer equilibrium exists in which the proposer's optimal proposal choice $\{p^*, T^*\}$ satisfies $p^* = V_{k^*}$ and $T^* = \lfloor \frac{N+k^*}{2} \rfloor$, for some integer $k^* \in \{-1, 0, \ldots, N\}$.*

Proposition 2 establishes the equilibrium existence and shows that the set of potential optimal prices and AoN targets is finite. To see this, notice that based on Proposition 1, for a given T, any $p \in (V_{k-1}, V_k]$ induces the same supporting decisions. The proposer always finds that $p = V_k$ strictly dominates any $p \in (V_{k-1}, V_k)$. We can therefore focus our analysis on $p \in \{V_{-1}, V_0, \ldots, V_N\}$. We exclude $k < -1$ because $V_{-1} = 1 - q$ is already sufficiently low to induce an UP cascade from the very beginning. Different from Proposition 1, it is possible to have multiple equilibria here because more than one k^* may generate the same maximum expected revenue for the proposer.

In equilibrium, the proposer chooses the optimal level of the AoN threshold jointly with price to maximize the expected revenue. The proposer is concerned about DOWN cascades because once they start, subsequent agents all reject. As shown in Proposition 1, there is no DOWN cascade before approaching the threshold. For a given equilibrium price V_{k^*}, a higher AoN target reduces the possibility of DOWN cascades or delays their arrival, so that the proposer can collect more proceeds conditional on implementation. Yet, a higher threshold itself is more difficult to reach. The proposer facing the tradeoff finds the optimal AoN threshold to be the one that is just large enough to exclude DOWN cascades that reduce total expected proceeds. The following corollary characterizing DOWN cascades proves useful for our discussion of model implications below.

COROLLARY 1 (Unidirectional Cascades): *With endogenous threshold implementation and contribution price, the necessary and sufficient condition for a DOWN cascade entails only the last agent ($i = N$) herding (i.e., ignoring the private signal and rejecting) and an implementation failure even when all private signals are aggregated publicly.*

Corollary 1 implies that for all practical purposes, all surviving DOWN cascades are of no concern here because they can start only from the last agent and do not affect project implementation. To see this, notice that when the price is $p = V_k$, if the DOWN cascade starts with agent N but not agent $N-1$, agent $N-1$ must be rejecting due to a negative signal instead of herding to reject despite having a positive signal. In that case, the public valuation before

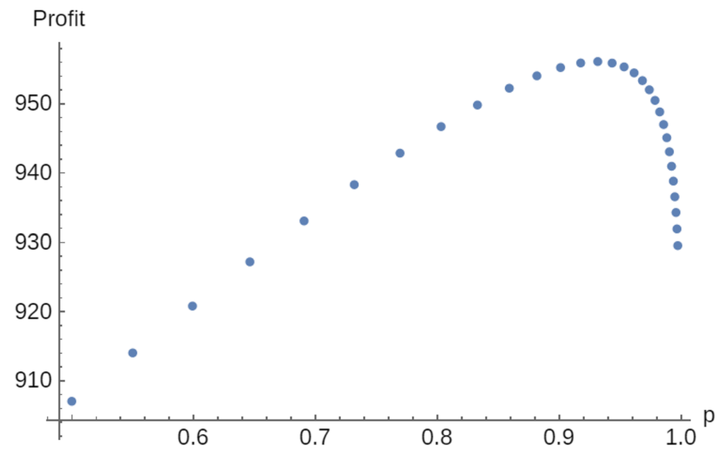


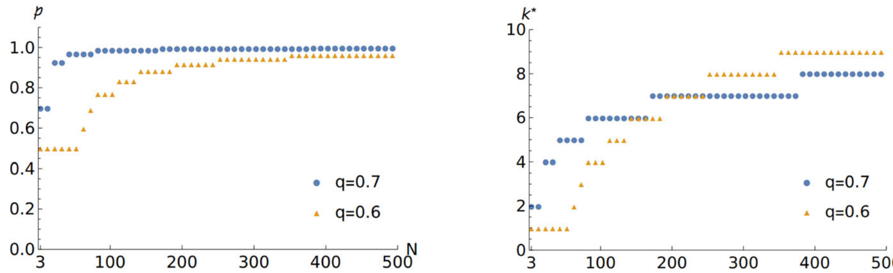
Figure 2. Proposal profit as a function of price. This figure plots simulated expected profit as a function of price for $N = 2,000$, $\nu = 0$, and $q = 0.55$. For each price V_k, the threshold is set as the optimal one given the price $T = \lfloor \frac{N+k}{2} \rfloor$. The optimal $k^* = 13$, and the profit-maximizing price is around 0.93. (Color figure can be viewed at wileyonlinelibrary.com)

agent $N-1$'s action cannot be less than V_{k-1}, as otherwise she would already start a DOWN cascade. Moreover, the public valuation just before agent N's action cannot be bigger than V_{k-2}, as otherwise she is not guaranteed to start a DOWN cascade.

Overall, we find that agent $N-1$ must observe a negative private signal $x_{N-1} = -1$, and the valuation of the project based on public information before her action is V_{k-1}. As such, even with all N private signals aggregated, the project has a maximum expected value of V_{k-1}, which is smaller than the price and consistent with a DOWN cascade in which the project is rejected by the last agent regardless of her private signal. Such a DOWN cascade does not affect the proposer's payoff much and has almost no impact on information aggregation. The unidirectional nature of potential cascades turns out to have important implications for project implementation and information aggregation, as we demonstrate in Section III.

We next turn to optimal pricing. Without an AoN threshold, that is, $T = 1$, the concern about DOWN cascades is so severe that in equilibrium the proposer avoids DOWN cascades by choosing a sufficiently low price to trigger an UP cascade at the very beginning (Lemma 1). Endogenous AoN thresholds mitigate this concern. The optimal pricing problem is now very similar to that in the auction literature: A higher price allows the proposer to extract more rent from each supporter, but is associated with a higher optimal AoN target, reducing the probability of implementation. Figure 2 illustrates how the entrepreneur's profit, which depends on implementation outcomes, varies with the optimal AoN threshold and thus its corresponding price.

Recall that in Lemma 1 without threshold implementation, the optimal price is *independent* of the number of agents when there is no AoN threshold. With



Panel A. Optimal price for $N = 3$ to 500. Panel B. Corresponding k^* for $N = 3$ to 500.

Figure 3. Cascades and optimal prices as N changes. This figure shows the numerical solution of optimal price as a function of N for $\nu = 0$. The blue plots represent the case in which $q = 0.7$, and the orange plots represent the case in which $q = 0.6$. Panel A plots the optimal price V_k, while Panel B shows the corresponding k^*. (Color figure can be viewed at wileyonlinelibrary.com)

AoN thresholds, the number of remaining agents and thus the total number of agents do affect k^*. Proposition 2 implies that the optimal price depends on the number of agents N. The following proposition shows that, in general, the optimal price p^* increases as N increases.

PROPOSITION 3: *The optimal price p^* depends on N and has a lower bound that is weakly increasing in N and approaches one as $N \to \infty$. In particular,* $\lim_{N\to\infty} p^*(N) = 1$.

A larger agent base implies a greater chance to learn about the project's quality. Because the probability of reaching a certain AoN target is higher for good projects, the concern about implementation failure is less pronounced, allowing the proposer to set the optimal price higher to increase expected proceeds. Figure 3 shows the optimal pricing for different values of N, where Panel A plots the absolute price level and Panel B plots the associated k^*. With an endogenous AoN threshold, the proposer can charge a higher price for a larger crowd, which can even appear "overpriced" ex ante, that is, $p > \mathbb{E}[V]$. Our findings on pricing are important because the under- or overpricing of securities or products may affect the success or failure of a project proposal and in turn the real economy (Welch (1992)). We discuss these model implications next.

III. Implications of Threshold Implementation and Large Crowds

Two key functions of modern financial markets and digital platforms are funding good projects and aggregating localized/decentralized information to inform investors' and policymakers' decisions. One feature that distinguishes crowdfunding platforms from venture capital lies in the large crowds they access. For example, according to Kickstarter's official statistics as of November 2020, the crowdfunding platform has 18.87 million total backers and the top 10 most popular projects have 74,410 to 219,380 backers, and the Crowdfunding

Center reports that fully funded projects have on average 300 backers.[14] In this section, we examine the immediate implications of AoN thresholds for project implementation and information aggregation as well as equilibrium outcomes when the crowd size becomes large.

We show that threshold implementations generally improve proposal feasibility, project selection, and information aggregation. In the limit of large crowds, we prove that project implementation and information aggregation become fully efficient—results not obtainable in earlier models of dynamic observational learning and crowdfunding.

A. Project Implementation

A financial marketplace serves to match capital with worthy projects. It is socially efficient to ensure that good projects and good projects only are financed.

A.1. Proposal Feasibility

Lemma 1 establishes an upper bound on the price in standard cascade models above which the proposal is infeasible, that is, good projects with production cost $\nu > q$ cannot be supported because even the break-even price triggers DOWN cascades. Threshold implementations allow proposers to charge $p > q$ and still implement the projects.

PROPOSITION 4: *Projects with $\nu > q$ cannot be implemented without AoN thresholds; all projects have a positive ex ante probability of being implemented under endogenous pricing and AoN thresholds.*

Obviously, projects with $\nu > V_N$ cannot be financed because agents' posterior valuation can never exceed V_N. Charging $p > \nu$ does not trigger a DOWN cascade if T is set to be sufficiently high. As a result, crowdfunding and the like with endogenous AoN thresholds or sufficiently high exogenous thresholds enable the financing of projects with high production costs for which funding would otherwise be infeasible (Welch (1992)). This result is consistent with Mollick and Nanda (2015), who empirically document that crowdfunding is more likely to finance projects with costly production that a group of experts would not finance in traditional settings.

A.2. Project Selection

Without threshold implementation, UP cascades start from the very beginning and all projects are implemented, resulting in a poor project selection (Welch (1992)). With AoN thresholds, DOWN cascades do not occur before reaching the implementation threshold, neither do UP cascades start from

[14] See, for example, https://www.statista.com/statistics/288345/number-of-total-and-repeat-kickstarter-project-backers/, https://www.statista.com/statistics/378054/most-backed-kickstarter-projects/, and https://www.thecrowdfundingcenter.com/data/projects.

the beginning. Thus, good projects have a higher chance of reaching the target threshold due to prolonged public information aggregation before any cascade.[15] Project selection therefore improves. In other words, given $q > \frac{1}{2}$ and since we focus on informer equilibria, good projects are more likely to be implemented than bad projects (i.e., $\Pr(A_N \geq T|V = 1) \geq \Pr(A_N \geq T|V = 0)$). We then have the following proposition.

PROPOSITION 5: *Let* $\mathcal{P}^I \equiv 1 - \Pr(A_N \geq T|V = 1)$ *and* $\mathcal{P}^{II} \equiv \Pr(A_N \geq T|V = 0)$. *Then,* $\lim_{N\to\infty} \mathcal{P}^I_N = \lim_{N\to\infty} \mathcal{P}^{II}_N = 0$.

Note that $\mathcal{P}^I$ and $\mathcal{P}^{II}$ correspond to the probabilities of missing a good project (Type I error) and financing a bad project (Type II error). While N does not matter in standard cascade models, threshold implementation links the timing and correctness of cascades to the size of the crowd. Both types of errors approach zero as N becomes large because a larger crowd implies a higher endogenous optimal price, which delays the arrival of UP cascades and improves the correctness of implementation. In general, while UP cascades do lead to some bad projects being financed, such Type II errors are not as frequent as in Welch (1992) and Lemma 1, in which all bad projects are financed with endogenous pricing and the probability of the cascade being correct is $\frac{1}{2}$ (i.e., uninformative). In our setting, the wisdom of the crowd is fully harnessed to distinguish good projects from bad ones. As for the allocation of surplus, investors' share vanishes in the limit because the price approaches the true value of a good project, and the proposer eventually gets all of the surplus from the project implementation.

B. Information Aggregation

Sequential support-gathering processes such as crowdfunding allow the market to aggregate investors' private signals and (partially) reveal the aggregated information to the public. Recent studies provide both theoretical arguments (Strausz (2017); Chemla and Tinn (2020); Brown and Davies (2020)) and empirical evidence that entrepreneurs use crowdfunding as an information aggregation mechanism (Mollick and Kuppuswamy (2014); Da Cruz (2018); Xu (2017)).[16]

The previous section on better project selection partially reflects better information aggregation. We now formalize the result on information aggregation,

[15] Unidirectional cascade and threshold implementation also imply that offerings in our setting can fail, in contrast to offerings never failing in Welch (1992). Our model is thus consistent with the possibility that some offerings fail occasionally and/or are withdrawn without having to invoke insider information as Welch (1992) does.

[16] Reduction of search and matching effort, divisibility of funding and low communication costs, greater outreach, decentralized participation, timely disclosure and monitoring, and so forth are generally recognized as key advantages of Internet-based platforms for aggregating information and harnessing wisdom from the crowd. In fact, the Securities and Exchange Commission (SEC) also recognizes in its final rule regulating crowdfunding that "individuals interested in the crowdfunding campaign – members of the 'crowd'...fund the campaign based on the collective 'wisdom of the crowd'" (Li (2017)).

which complements but differs from project selection.[17] In particular, information aggregation generally improves even in the event of crowdfunding failure.

PROPOSITION 6: *Crowdfunding, successful or not, is informative of project quality:* $\mathbb{E}[V|\mathcal{H}_N]$ *is weakly increasing in* A_N *and* $\mathbb{E}[V|\mathcal{H}_N, A_N < T^*]$ *is strictly increasing in* A_N.

Different from standard cascade models with DOWN cascades conditional on failing to reach the AoN threshold, the proposer updates beliefs more positively with more supporting agents. Our model has the natural implication that the belief updates on V based on incremental support are smaller, conditional on project implementation, because they likely involve an UP cascade and information aggregation is more limited. This is consistent with Xu (2017), who finds that conditional on fundraising success, a 50% increase in pledged amount leads to a 9% increase in the probability of commercialization outside the crowdfunding platform—a small sensitivity of the project prospects to changes in the level of support.

Note that we achieve full information aggregation in the large-crowd limit as a direct consequence of Proposition 5. Because good projects and good projects only are funded in the limit, one knows $V = 1$ or $V = 0$ based on the implementation outcome by the end of the crowdfunding, which is public information.

IV. Discussion and Extensions

A. Option to Wait

Potential crowdfunding contributors often cannot or do not wait because of nontrivial attention or monitoring costs. Moreover, because the shares or products sold are often in limited supply, waiting may lead an agent to miss out on the opportunity. That said, in some cases, agents may choose to wait for more information. We now extend the model by expanding agents' action space to $a_i^t \in \{-1, 0, 1\}$, where zero indicates that agent i delays her decision in period t to the next period. Agents can continue waiting until the last period. In any period t, after agent t's decision, all agents waiting from earlier periods make decisions one at a time (ordered by their first arrival time). We stick with the tie-breaking rules above in this extension. Section I of the Appendix contains a formal description of the extension. The option to wait results in potential equilibrium multiplicity due to the coordination problem on waiting decisions and off-equilibrium beliefs. Nevertheless, the equilibrium characterized in Proposition 1 survives in a slightly modified form.

[17] Project selection discussed earlier corresponds to $\Pr(A_N \geq T|V = 1) \geq \Pr(A_N \geq T|V = 0)$, which is not the same as good projects being more likely to have a higher A_N. We can show that $V = 1$ does indeed lead to higher A_N in expectation, but it is equivalent to Proposition 6 (a higher A_N indicates that the project is more likely to be good) only under the common prior belief on V that $V = 0$ and $V = 1$ with equal probability. In general, project selection and information aggregation results differ under Bayesian updates.

PROPOSITION 7: *For any given (p, T), a subgame-perfect equilibrium exists in which those agents who would reject under Proposition 1 now delay their actions as much as possible while those who would support under Proposition 1 support upon their first opportunity to do so.*

Proposition 7 shows that given (p, T), the option to wait does not affect information aggregation because a decision to wait reveals a negative signal. In the case of an UP cascade, no one wants to deviate to wait. Now suppose there is no cascade yet. Agents with positive signals will support because supporting always dominates rejection and thus there is no need to wait in this equilibrium. For agents with negative signals, however, waiting until the end weakly dominates rejection and hence they wait. The next proposition shows that with endogenous (p, T), our finding on project implementation with large crowds is robust to options to wait.

PROPOSITION 8: *When agents can wait, there exists a sequence of equilibria as $N \to \infty$ in which the optimal price $p_N^* \to 1$. In the limit, all agents support good projects and good projects only, that is, the first-best project implementation is obtained.*

The intuition for this result is similar to that for Corollary 3. The absence of DOWN cascades helps investors avoid missing good projects and the high price screens out bad projects whose valuation cannot be sufficiently high as information arrives and gets aggregated. Notably, because agents with negative signals who wait can still invest or support later to not miss out on good projects, the project's scale reaches full efficiency in the limit, which is the first-best outcome not achievable without agents' option to wait.

B. Investor Heterogeneity and Dollar Amount Thresholds

According to the Crowdfunding Center, successful campaigns rely on large numbers of comparable, small contributions instead of concentrated, large contributions. Therefore, in the baseline model, the homogeneous contribution amount and threshold specification in the number of supporters reasonably balance tractability and realism. Nevertheless, many crowdfunded projects offer multiple contribution levels and require that a minimum dollar amount be met to be feasible. We next extend the model to illustrate the effects of investor heterogeneity in wealth and of implementation thresholds involving dollar amounts. We demonstrate a novel phenomenon of "prolonged learning" through partial support, which derives from a hitherto absent "funding gap versus belief gap" tradeoff. Sections I and IV of the Internet Appendix formalize the key insights and provide numerical procedures for designing optimal AoN thresholds under investor heterogeneity and dollar amount thresholds.

EXAMPLE 1: Suppose there are 30 agents. Each agent can choose $\{H, L, 0\}$, where $H = 1$ and $L = 0.3$ are contribution levels that the entrepreneur specifies. Poorer agents can only afford L. The project payoff is still $V = 0$ or 1 per dollar contribution. The AoN target is $T = 10.1$. Now consider a history such that $A_{i-1} = 9, k_{i-1} = \bar{k}(p) - 3$. Suppose $x_i = 1$. Should a rich agent i fully support ($a_i = H$), partially support ($a_i = L$), or reject ($a_i = 0$)?

Agent i understands that she gets nothing if she rejects. If she chooses to fully support with H, then $A_i = A_{i-1} + 1 = 10$ and $k_i = k_{i-1} + 1 = \bar{k}(p) - 2$. In equilibrium, even if the next three agents (i.e., $i+1, i+2, i+3$) all receive positive signals, agent $i+1$ (without knowing subsequent agents' signals) chooses not to support and starts a DOWN cascade. Consequently, agent i gets zero payoff. However, if agent i chooses partial support L, then $A_i = A_{i-1} + 0.3 = 9.3$ and $k_i = k_{i-1} + 1 = \bar{k}(p) - 2$. If the next three agents are poor but all receive positive signals and are of type L, then we have $a_{i+1} = a_{i+2} = a_{i+3} = L = 0.3$, $k_{i+3} = \bar{k}(p) + 1$, and $A_{i+3} = 10.2 > 10.1 = T$, in which case the project will be implemented and agent i receives a positive profit.

In this example, considering all of the options, partial support ($L = 0.3$) evidently can be preferred because it generates a positive expected payoff. Hence, it would be naive to conclude that one always contributes to the full extent possible if one supports, or that a wealthier agent never makes a low contribution. As we show in the Internet Appendix, in equilibrium, agents with negative signals refrain from contributing. But with positive signals, the example reveals that it could be optimal for a rich agent to switch their contribution from H to L to prolong learning at the expense of slower fundraising.

When the funding gap (the additional support needed to reach implementation) is small but the belief gap (the distance to the break-even valuation) is still large, an agent uses partial support to create "prolonged learning" because partial support allows for more rounds of trials before triggering implementation or a DOWN cascade. Such episodes of prolonged learning may occur multiple times before the agent eventually returns to full support or a DOWN cascade takes place, depending on whether the break-even belief (captured by $\bar{k}(p)$ in the baseline model) is reached. This tradeoff between reaching the implementation threshold and allowing for more observational learning can affect the ex ante information design of the crowdfunding, which constitutes an interesting topic for future research.

Moreover, in the Internet Appendix, we demonstrate that with variable dollar amount contributions, the entrepreneur can no longer extract the full surplus even as N goes to infinity. To extract the full surplus, the proposer needs to set prices such that all agents are indifferent between supporting and rejecting. This is problematic here because if the price is high enough to approach $V_{\bar{k}}$ (a necessary condition for full surplus extraction) and the gap between H and L is too big, then an agent will switch from high support to low support to prolong the campaign and information aggregation. As such, there exist some signal sequences such that an UP cascade is more likely to happen, which generates a positive payoff to the agents. Note that in an UP cascade, the agent with a positive signal must be getting some positive payoff, because she also supports the project when her signal is negative, which generates a lower but still nonnegative payoff.

Note that similar to adding options to wait, enlarging agents' action space augments the communication space as well. Consequently, multiple equilibria may exist. For example, with small p and N but a relatively large T, the threshold is sufficiently far away that a contribution L would still allow sufficient subsequent signal aggregation. Yet, as long as contributing L would not mislead subsequent agents too much, the equilibrium belief is that both rich agents with negative signals and poor agents with positive signals will contribute L, and the proportion of rich agents is sufficiently large. This way, rich agents with negative signals can still be protected from below while not missing out on the investment. We leave a more comprehensive characterization of various equilibria to future work.

C. Free Riders and Characterization of All Equilibria

We can characterize PBNEs satisfying Assumptions 1 and 2 even beyond informer equilibria—a daunting task that most models of observational learning leave out. We first show that all possible equilibria involve a group of "informers" and a group of "free riders" whose actions before a cascade are ignored in equilibrium. Mathematically, agent i is considered a free rider if $\mathbb{E}[V|\mathcal{H}_{i-1}] = \mathbb{E}[V|\mathcal{H}_i] < V_{\bar{k}(p)+1}$ before any UP cascade. In other words, following subhistory $\mathcal{H}_{i-1}$, it is common knowledge that subsequent rational agents would not update their beliefs based on agent i's action, even though an UP cascade has not started yet.

Free riding differs from information cascades. Although an agent's action is uninformative as in information cascades, agents can still take informative actions after the free rider's move, and information aggregation resumes until a cascade starts, another free rider appears, or the game ends. We call an equilibrium with a positive number of free riders a free-rider equilibrium. To give an example, suppose $\nu < \frac{1}{2}$, $p = \frac{1}{2} = \frac{q^0}{q^0+(1-q)^0}$, and the target is $T = N$. Then, there is a subgame free-rider equilibrium in which all agents but the N^{th} agent support regardless of their private signal, and the N^{th} agent supports if and only if $x_N = 1$.

In a free-rider equilibrium, which agents become free riders is generally path-dependent (i.e., specific to sequential realizations of private signals). Whether an agent becomes a free rider depends on subsequent agents' higher order beliefs. As in the informer equilibrium, such a phenomenon does not exist in conventional models because absent threshold implementation, the agent's expected payoff at the time of decision making is independent of subsequent agents' actions.

LEMMA 2: *Under Assumption 1 for tie-breaking, a PBNE is either an informer equilibrium or a free-rider equilibrium. For $p \in \{V_k, k = -1, 0, \dots N\}$, all free-rider subgame equilibria are weakly Pareto-dominated by the informer subgame-perfect equilibrium described in Proposition 1. Free-rider subgame equilibria involving at least two free riders are strictly Pareto-dominated.*

To analyze the large-crowd limit, we use a standard equilibrium selection based on payoff dominance (Harsanyi and Selten (1988), which can be motivated in our context by communications before agents receive signals) to focus on *Pareto-undominated* subgame equilibria. This refinement rules out nuisance equilibria such as the example given before Lemma 2 in which investors coordinate on Pareto inferior outcomes, but still allows the large class of free-rider equilibria for general $p \notin \{V_k, k = -1, 0, \dots N\}$.[18] Under the refinement, when $p \in \{V_k, k = -1, 0, \dots N\}$, we only need to consider informer equilibria and free-rider equilibria with one free rider. This allows us to show that the number of informers is unbounded as N increases and that informer and Pareto-undominated free-rider equilibria deliver qualitatively the same results as $N \to \infty$.

PROPOSITION 9: *In any sequence of endogenous designs $\{p(N), T(N)\}_{N=1}^{\infty}$ and Pareto-undominated subgame equilibria, as $N \to \infty$, $p^*(N) \to 1$, good projects and good projects only are implemented, and public posterior valuation converges to the true quality V.*

The proposition implies that no matter which equilibrium is selected among all the Pareto-undominated ones, in the limit the proposer charges a high enough price to preclude DOWN cascades and ensure both efficient project implementation and full information aggregation. In the proof, we actually show that for large N, implementation efficiency and information aggregation generally improve relative to those in standard information cascade settings without threshold implementations. Given that financing projects and aggregating information are arguably the most important functions of financial markets, the impact of threshold implementation, especially with large crowds, cannot be overstated.

V. Conclusion

We incorporate AoN threshold implementation into a standard model of information cascades and find that agents' payoff interdependence results in unidirectional cascades in which agents rationally ignore private signals and imitate prior agents only if the prior agents decide to support. Information aggregation, proposal feasibility, and project selection all improve. As the number of agents approaches infinity, equilibrium project implementation and information aggregation achieve socially efficient levels despite information frictions. These findings add to theories of observational learning and dynamic contribution games, as well as to the emerging literature on entrepreneurial crowdfunding and FinTech platforms.

An important implication of our model is that digital funding platforms can help entrepreneurs access a larger agent base to better harness the wisdom of the crowd than traditional funding channels, as envisioned by

[18] Note that nuisance equilibria can also be ruled out by considering agents' option to wait. Obviously, every agent observing signal $x_i = -1$ is better off waiting.

regulatory authorities. We show that specific features and designs such as endogenous AoN thresholds are crucial to capitalizing on these potential benefits, especially for sequential sales in the presence of informational frictions. For parsimony and generality, we leave out some application-specific details. For instance, third-party certification can impact crowdfunding outcomes (Knyazeva and Ivanov (2017)). In addition, a project proposer may price discriminate or control the information flow to potential investors. Incorporating these institutional features and jointly considering information and mechanism designs constitute promising future research topics.

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Appendix

A. Proof of Lemma 1

PROOF: For agent 1, her posterior belief after observing $x_1 = 1$ is $\mathbb{E}[V|x_1 = 1] = q$. If $p > q$, then agent 1 chooses rejection regardless of her private signal, and a DOWN cascade starts from the beginning for sure. We thus have the first part of the lemma.

Similarly, $p = 1 - q = \mathbb{E}[V|x_1 = -1]$ induces an UP cascade starting from the beginning for sure, and hence the entrepreneur or proposer has no incentive to charge $p < 1 - q$. Therefore, $p \in [1 - q, q]$. Following Bikhchandani, Hirshleifer, and Welch (1992), the common posterior belief of Bayesian agents follows

$$V_k = \frac{q^k}{q^k + (1-q)^k}, \qquad k \in \mathbb{Z}, \tag{A.1}$$

where k is the difference between the number of inferred positive signals and the number of inferred negative signals. For each $p \in (V_{k-1}, V_k]$, $p = V_k$ induces exactly the same decisions and, in turn, the same number of supporting agents, so in equilibrium the proposer always charges $p^* = V_k$ for some $k \in \{-1, 0, 1, \ldots, N\}$. Consequently, only three prices are possible: $1 - q$, $\frac{1}{2}$, and q. Let $\Pi(p, N)$ be the expected profit when the price is p and there are $N \geq 2$ potential agents. Without AoN thresholds, $\Pi(p, N)$ is obviously increasing in N. Next, we examine the three possible optimal prices and show that $p = 1 - q$ dominates.

(i) $p = 1 - q$: The total profit is $(1 - q)N$ when $\nu = 0$.

(ii) $p = \frac{1}{2}$: After the first two observations, $(x_1, x_2) = (-1, -1)$ induces a DOWN cascade yielding zero profit, $(1, -1)$ and $(1, 1)$ both induce a UP cascade at agent 1 due to the tie-breaking assumption, which leads to an expected payoff of $\frac{q+(1-q)}{2}\frac{1}{2}N$, and $(-1, 1)$ does not change subsequent agents' belief. Therefore, $\Pi(\frac{1}{2}, N) = \frac{q+(1-q)}{2}\frac{1}{2}N + \frac{q(1-q)+(1-q)q}{2}(\frac{1}{2} +$

$\Pi(\frac{1}{2}, N-2)) \leq \frac{1}{4}N + (1-q)q(\frac{1}{2} + \Pi(\frac{1}{2}, N))$. Thus, $p = \frac{1}{2}$ is dominated by $p = 1-q$ if

$$\Pi(p, N) \leq \frac{\frac{N}{4} + \frac{(1-q)q}{2}}{1-(1-q)q} \leq (1-q)N \ for \ N = 2, 3, \ldots, \tag{A.2}$$

which holds for $q \in (\frac{1}{2}, \frac{3}{4} + \frac{1}{4}(3^{\frac{1}{3}} - 3^{\frac{2}{3}})]$.

(iii) $p = q$: After the first two observations, $(1, 1)$ induces an UP cascade, $(-1, -1)$ and $(-1, 1)$ both induce a DOWN cascade after agent 1, and $(1, -1)$ does not change the belief. The expected profit is $\Pi(p, N) = \frac{(1-q)^2+q^2}{2}qN + \frac{q(1-q)+(1-q)q}{2}(q + \Pi(p, N-2)) \leq \frac{(1-q)^2+q^2}{2}qN + (1-q)q(q + \Pi(p, N))$. Thus, $p = q$ is dominated by $p = 1-q$ if

$$\Pi(p, N) \leq \frac{\frac{(1-q)^2+q^2}{2}qN + q^2(1-q)}{1-(1-q)q} \leq (1-q)N \ for \ N = 2, 3, \ldots. \tag{A.3}$$

One can verify that the inequality holds for $q \in (\frac{1}{2}, \frac{3}{4} + \frac{1}{4}(3^{\frac{1}{3}} - 3^{\frac{2}{3}})]$. □

B. Proof of Proposition 1

PROOF: We first analyze the dynamics of common posterior belief given the truth-telling contribution strategy before any cascades. Notice that given truth telling, when there is no cascade yet, k_i, the difference between the number of inferred positive signals and the number of inferred negative signals, is the same as the difference between the number of private positive signals and the number of negative private signals. Then, as in Bikhchandani, Hirshleifer, and Welch (1992), the common posterior belief of Bayesian agents follows

$$V_{k_i} = \frac{q^{k_i}}{q^{k_i} + (1-q)^{k_i}}, \qquad k \in \mathbb{Z}. \tag{A.4}$$

We prove the result by induction on the length of the history $i \in \{0, 1, \ldots, N\}$, where $\mathcal{H}_0 = \varnothing$. For $i = 0$, $\mathbb{E}[V|\mathcal{H}_0] = \frac{1}{2} = V_0$.

Suppose the statement is true of all histories $\mathcal{H}_l (l \leq i)$, that is, $\mathbb{E}[V|\mathcal{H}_l] = V_{k_l}$. Now consider a history $\mathcal{H}_{i+1}$. If there is no cascade yet, then Bayes' rule implies

$$\begin{aligned}
\mathbb{E}[V|\mathcal{H}_{i+1}] &= \Pr(V = 1|\mathcal{H}_{i+1}) = \Pr(V = 1|a_{i+1}, \mathcal{H}_i) \\
&= \frac{\Pr(\mathcal{H}_i)\Pr(V = 1|\mathcal{H}_i)\Pr(a_{i+1}|V = 1, \mathcal{H}_i)}{\sum_{j\in\{0,1\}}\Pr(\mathcal{H}_i)\Pr(V = j|\mathcal{H}_i)\Pr(a_{i+1}|V = j, \mathcal{H}_i)} \\
&= \frac{V_{k_i}\Pr(a_{i+1}|V = 1, \mathcal{H}_i)}{V_{k_i}\Pr(a_{i+1}|V = 1, \mathcal{H}_i) + (1 - V_{k_i})\Pr(a_{i+1}|V = 0, \mathcal{H}_i)} \\
&= \frac{q^{k_{i+1}}}{q^{k_{i+1}} + (1-q)^{k_{i+1}}}.
\end{aligned} \tag{A.5}$$

If there is a cascade, then by definition the public does not infer anything from agent $i+1$'s action, $k_{i+1} = k_i$, and $\mathbb{E}[V|\mathcal{H}_{i+1}] = V_{k_i}$.

Given the results above, we formally characterize the equilibrium contribution strategy and the evolution of the public posterior as follows:

$$a_i^* = \begin{cases} x_i & \text{if } \; A(\mathcal{H}_{i-1}) < T-1 \;\;\&\;\; k_{i-1} \leq \bar{k}(p) \\ 1 & \text{if } \; k_{i-1} > \bar{k}(p) \\ -1 & \text{if } \; A(\mathcal{H}_{i-1}) \geq T-1 \;\;\&\;\; k_{i-1} < \bar{k}(p)-1 \\ x_i & \text{if } \; A(\mathcal{H}_{i-1}) \geq T-1 \;\;\&\;\; k_{i-1} \in \{\bar{k}(p), \bar{k}(p)-1\} \end{cases}, \tag{A.6}$$

$$k_i = \begin{cases} k_{i-1} + a_i^* & \text{if } \; A_{i-1} < T-1 \;\;\&\;\; k_{i-1} \leq \bar{k}(p) \\ k_{i-1} & \text{if } \; k_{i-1} > \bar{k}(p) \\ k_{i-1} & \text{if } \; A_{i-1} \geq T-1 \;\;\&\;\; k_{i-1} < \bar{k}(p)-1 \\ k_{i-1} + a_i^* & \text{if } \; A_{i-1} \geq T-1 \;\;\&\;\; k_{i-1} \in \{\bar{k}(p), \bar{k}(p)-1\}. \end{cases} \tag{A.7}$$

Notice that by definition, $A_0 = k_0 = 0$ to match the prior. We call the agent observing $T-1$ preceding supporters the "gatekeeper" for all preceding supporters because her decision affects whether other supporters incur the contribution cost and receive the project payoffs. In equilibrium, agents are truth telling and the public posterior updates before any cascades. To prove that the characterization in Proposition 1 constitutes a PBNE, we first state and prove Lemma A.1—the expected value of the adoption is bounded above by $V_{\bar{k}(p)+1}$. In fact, an UP cascade starts once a strong expectation is formed and it blocks further learning by subsequent agents. Anticipating this behavior from subsequent agents, early agents with a negative signal do not support the proposal if an UP cascade has not started yet. This makes the support of the agents with a positive signal informative for the subsequent agents. For the histories that either the AoN target or a cascade has been reached, the proof is trivial.

LEMMA A.1: *Suppose $\bar{k}(p) \geq -1$. Then, in every equilibrium, when it is still possible to reach the AoN target T, the following relation holds for every $2 \leq i \leq N$:*

$$\mathbb{E}[V|\mathcal{H}_{i-1}] \leq V_{\bar{k}(p)+1}. \tag{A.8}$$

In other words, there is an upper bound on the expected value of the project as a function of p.

PROOF: Suppose the contrary. Then, there exists an agent i with $\mathbb{E}[V|\mathcal{H}_i] = V_{\bar{k}(p)+2}$ and $\mathbb{E}[V|\mathcal{H}_{i-1}] = V_{\bar{k}(p)+1}$. However, given $\mathbb{E}[V|\mathcal{H}_{i-1}] = V_{\bar{k}(p)+1}$, i would accept the proposal regardless of her private signal because

$$\mathbb{E}[V|x_i, \mathcal{H}_{i-1}, A_N \geq T] \geq \mathbb{E}[V|x_i, \mathcal{H}_{i-1}] \geq \mathbb{E}[V|x_i = -1, \mathcal{H}_{i-1}] = V_{\bar{k}(p)} \geq p. \tag{A.9}$$

Recall that all expectations are conditional on equilibrium strategies of other agents, and A_N is the total number of supporters among all agents. The first inequality (from left) in (A.9) follows from the fact that the gatekeeper makes her decision based on an information set that fully nests $\mathcal{H}_{i-1}$. Therefore, her support positively updates agent i's belief.

Equation (A.9) shows that agent i's action is not informative for subsequent agents. Thus, $\mathbb{E}[V|\mathcal{H}_i] = \mathbb{E}[V|\mathcal{H}_{i-1}] = V_{\bar{k}(p)+1}$, a contradiction. □

We are now ready to prove the equilibrium characterization. For notational ease, we replace $\bar{k}(p)$ by $\bar{k}$. We proceed by examining the optimal strategy for different histories.

$A_{i-1} < T - 1$ and $k_{i-1} \leq \bar{k}$.

According to (2), the agent chooses to support if and only if $\mathbb{E}[V|x_i, \mathcal{H}_{i-1}, A_N \geq T] \geq p$. We examine two cases $x_i = 1$ and $x_i = -1$ separately.

(i) $x_i = 1$: If agent i chooses to reject, she receives zero. If agent i supports, then consider a history $\mathcal{H}_N \succ \mathcal{H}_i = (\mathcal{H}_{i-1}, 1)$ in which the proposal is accepted. Denote the gatekeeper by g, that is, g is the smallest integer such that $A_g = T$. Note that g is a random variable depending on the history $\mathcal{H}$. Since the subsequent agents $\{i+1, i+2\ldots\}$ perfectly infer $x_i = 1$ from the support of agent i, agent g's information set fully nests that of agent i. We therefore get

$$\begin{aligned}\mathbb{E}[V|x_i, \mathcal{H}_{i-1}, A_N \geq T] &= \mathbb{E}[V|x_i, \mathcal{H}_{i-1}, \mathbb{E}[V|x_g, \mathcal{H}_{g-1}] \geq p] \\ &= \mathbb{E}[V|\mathbb{E}[V|x_g, \mathcal{H}_{g-1}] \geq p] \\ &\geq p.\end{aligned}$$

In other words, as long as it is possible to implement the project, it is optimal for agent i to support.

(ii) $x_i = -1$: In this case, in equilibrium agent i's support would be misinterpreted by subsequent agents as a positive signal, that is, $k(\mathcal{H}_i) = k(\mathcal{H}_{i-1}) + 1$, while the correct posterior should be $k(\mathcal{H}_i) = k(\mathcal{H}_{i-1}) - 1 = k(\mathcal{H}_{i-1}) + 1 - 2$. Moreover, if the proposal is accepted, Lemma A.1 implies $\mathbb{E}[V|x_g, \mathcal{H}_{g-1}] \in \{V_{\bar{k}}, V_{\bar{k}+1}\}$. Therefore, agent i, knowing that her signal is incorrectly inferred by her action, assigns an expected value bounded above by $V_{\bar{k}-1} < p$ conditional on her signal and project implementation. She thus optimally chooses $a_i = -1$, lest she receives a negative expected payoff.

In sum, when $k_{i-1} \leq \bar{k}$ and $A_{i-1} < T - 1$, agent i follows her private signal and the subsequent agents update their beliefs accordingly.

$A_{i-1} \geq T - 1$ and $k_{i-1} \leq \bar{k}$.

In this case, the project is implemented once agent i supports, and agent i's problem is the same as in standard cascade models. Taking that into account, agent i supports the project if and only if $\mathbb{E}[V|x_i, \mathcal{H}_{i-1}] = V_{k_{i-1}+x_i} \geq p$. The only time the strategy is separating is when $k_{i-1} \in \{\bar{k}, \bar{k}-1\}$. Given this observation, it is easy to check that the strategies specified in (A.6) are optimal in this case.

$k_{i-1} > \bar{k}$.

Clearly, $\mathbb{E}[V|x_i, \mathcal{H}_{i-1}] \geq V_{\bar{k}} \geq p$. In other words, agent i gains at least $V_{\bar{k}} - p$ in expectation if the AoN target is reached. It is therefore not profitable to reject regardless of agent i's type, which proves optimality in this case.

Finally, we show that the equilibrium strategy profile is the unique informer equilibrium. Our first step is to show that in an informer equilibrium, all strategies are separating before reaching a cascade and the AoN threshold is possible to reach. To do so, we show that there are no partially separating strategies because of the tie-breaking rule in Assumption 1. When $N - (i+1) < T - A_i - 1$, it is impossible to reach the AoN target, and Assumption 1 suggests a fully separating strategy. We now consider scenarios in which it is still possible to reach the AoN target (i.e., $N - (i+1) \geq T - A_i - 1$). Because there is not a cascade yet, Agent i's payoff satisfies $E(V|\mathcal{H}_{i-1}, x_i = 1, a_i = 1, A_N \geq T) \geq p > E(V|\mathcal{H}_{i-1}, x_i = -1, a_i = 1, A_N \geq T)$, and hence agent i has no incentive to play a partial pooling strategy.

Therefore, an informer equilibrium must be an equilibrium such that all strategies are fully separating before reaching a cascade and the AoN threshold is possible to reach. To show uniqueness, we need only to show that a cascade cannot occur when $A_{i-1} < T - 1$ and $k_{i-1} \leq \bar{k}$ (and the AoN threshold is possible to reach).[19] Suppose to the contrary that such a cascade exists in an equilibrium. Then, agents should choose the same action after such a history. First, it is not possible that all agents support if doing so leads to project implementation, because the gatekeeper would certainly reject if she has a negative signal, this would contradict the assumption that the gatekeeper is part of the cascade. Second, it is not possible that all agents reject because this would lead to the eventual rejection of the proposal. In this case, all agents are indifferent between supporting and not supporting, and their rejections violate Assumption 1. The contradiction implies that the equilibrium is unique.

The characterization of UP cascades and DOWN cascades follows directly. □

C. Proof of Proposition 2

PROOF: Without loss of generality, we focus on cases in which $\nu \leq V_N$. We prove the proposition via multiple steps and lemmas. In Step 1, we show that the optimal price p^* must be in the set $\{V_{-1}, V_0, \ldots, V_N\}$. The optimal pair (p^*, T^*) therefore belongs to set $\{(p, T)|p \in \{V_{-1}, \ldots, V_N\}, T \in \{1, \ldots, N\}\}$ with finite number of elements, which ensures the existence of a solution to the proposer's optimization problem. In Steps 2 to 6, we show that T^* is the optimal AoN threshold because it is a dominant choice. We discuss all $T > T^*$ and $T < T^*$, in Steps 2 and 3, respectively, when $k^* > 0$. For $k^* = 0$, we prove the

[19] When $A_{i-1} \geq T - 1$, the subsequent actions are irrelevant for agent i's payoff, and her action depends only on the price and the expected value given x_i and $\mathcal{H}_{i-1}$. Therefore, the optimal strategy is the same across all equilibria (subject to Assumption 1).

optimality of T^* for $q(1-q) > \frac{1}{6}$ and $q(1-q) \leq \frac{1}{6}$, in Steps 4 and 5, respectively. In Step 6, we finish the proof by showing that T^* is optimal when $k^* = -1$.

Step 1: Optimality of p^* and Existence of the Solution

First, it is straightforward to see that $p \geq \nu$, because any equilibrium price $p < \nu$ would generate a negative return for the proposer and is strictly dominated by $p = \nu$. We next show that $p^* \in \{V_{-1}, V_0, \ldots, V_N\}$. Note that all $p < V_{-1}$ are suboptimal since an UP cascade starts from the very first agent and all agents would support if $p \leq V_{-1}$. Moreover, clearly, the posterior of the agents never exceeds V_N, and thus all $p > V_N$ are suboptimal as well. We therefore focus on the case $p \in [V_{-1}, V_N]$.

For any $p \in (V_{k-1}, V_k]$, $k \in \{0, 1, \ldots, N\}$, in the subgame, agents follow the equilibrium strategy profile specified in (A.6), as it depends only on $\bar{k}(p)$ and T. This implies that any choice of $p \in (V_{k-1}, V_k)$ induces the same $\bar{k}(p)$ and is dominated by $p = V_k$, for $k \in \{0, 1 \ldots N\}$. Consequently, $p^* \in \{V_{-1}, V_0, \ldots, V_N\}$. Notice that the set of T^* is also finite as $T \in \{1, 2, \ldots, N\}$. The proposer then need only to choose from a finite set of pairs (p, T), which guarantees the existence of the solution.

Notice that we cannot rule out multiple equilibria because it is possible to have multiple p^* (and corresponding V_{k^*}) that generate the same expected proceeds.

Some Helpful Definitions and Results

For the rest of the proof, given the equilibrium price V_{k^*}, we say that a sequence of signals $x \in X$ is "*T-supported*" for some $1 \leq T \leq N$ if the proposal is accepted for pair (V_{k^*}, T). Let $\tilde{X}_{T-1/T}$ be the set of all sequences of signals that are $(T-1)$-supported, but not T-supported, and let $\tilde{X}_{T/T-1}$ be the set of sequences of signals that are T-supported, but not $(T-1)$-supported (we provide an example of such sequences in the proof below). Recall that the optimal pair is (p^*, T^*), where $p^* = V_{k^*}$ and $T^* = \lfloor \frac{N+k^*}{2} \rfloor$. The following two lemmas are useful for our analysis.

LEMMA A.2: *Suppose sequence $x \in X$ is $(T-1)$-supported and not T-supported, for some $T \leq T^*$. Then, there are at least $T-1$ positive signals in x. Furthermore, if h_{T-1} is the agent with the $(T-1)^{th}$ positive signal in the queue, then $h_{T-1} \leq N-2$ and $(x_{h_{T-1}}, x_{h_{T-1}+1}, x_{h_{T-1}+2}) = (1, -1, -1)$.*

PROOF: First, we show that if a sequence $x' \in X$ induces an UP cascade, then more than T^* agents support the proposal. To see this, suppose an UP cascade starts after Agent $r < N$, which implies $\sum_{i=1}^{r} x_i = k^* + 1$. One can then obtain that among those r agents, there are $\frac{r+k^*+1}{2}$ supporting agents. Therefore, since all the subsequent agents support the proposal, the total number of supporters is

$$\frac{r + k^* + 1}{2} + N - r > \frac{N + k^* + 1}{2} > T^*.$$

Given this result, since x is not T-supported, the AoN target $T-1$ cannot be reached by an UP cascade, and the existence of at least $T-1$ positive signals is necessary.

To see $h_{T-1} \leq N-2$, note that when the AoN target is $T-1$ and x is $T-1$-supported, the support of h_{T-1} requires that the following condition hold:

$$T - 1 - (h_{T-1} - (T-1)) \geq k^* \Rightarrow h_{T-1} \leq 2(T^* - 1) - k^*$$
$$\leq 2\left\lfloor \frac{N+k^*}{2} \right\rfloor - 2 - k^* \leq N - 2.$$

The only claim left to show is that $(x_{h_{T-1}}, x_{h_{T-1}+1}, x_{h_{T-1}+2}) = (1, -1, -1)$. This relation results from the assumption that x is not T-supported. It therefore implies that both $\sum_{i=1}^{h_{T-1}+1} x_i$ and $\sum_{i=1}^{h_{T-1}+2} x_i$ should be strictly less than k^*. The result is straightforward from the observation that $\sum_{i=1}^{h_{T-1}} x_i = k^*$. □

LEMMA A.3: *Suppose sequence $x \in X$ is $(T-1)$-supported, for some $T \leq T^*$. Then there exists an injective function $x'(x)$ that maps each sequence x to a distinct sequence $x'(x)$ such that $x'(x)$ is T-supported. The number of supporting agents in $x'(x)$ for T is weakly higher than that in x for $T-1$.*

PROOF: The proof of Lemma A.2 shows that if a sequence is both $(T-1)$-supported and T-supported, then reconsider $x'(x) = x$ and the number of supporting agents in both cases is the same. It remains to prove the lemma for any signal that is $(T-1)$-supported but not T-supported. From Lemma A.2, every sequence of signals $x \in \tilde{X}_{T-1/T}$ can be rewritten as $x = (\underbrace{\ldots}_{h_{T-1}-1}, 1, -1, -1, \ldots)$.

There exists a corresponding sequence $x' = (\underbrace{\ldots}_{h_{T-1}-1}, -1, 1, 1, \ldots) \in \tilde{X}_{T/T-1}$, in which only the three middle signals are reversed. Based on results in Proposition 1, there are exactly $T-1$ supporters in x (since a DOWN cascade starts at agent $h_{T-1}+2$), while there are at least T supporters in $x'(x)$. By construction, each x has a distinct image $x'(x)$, so $x'(x)$ is an injective function. □

The next couple of steps show that the optimal AoN threshold is $T^* = \lfloor \frac{N+k^*}{2} \rfloor$, given $p^* = V_{k^*}$.

Step 2: The Proof of $\pi(p^*, T) < \pi(p^*, T^*)$, for $T > T^*$ and $k^* > 0$

We simply show that for any $T > T^*$, if a sequence of signals is T-supported, then it is T^*-supported and has at least the same number of supporters as well. We next show that there exists at least one sequence that is T^*-supported but not T-supported.

To prove the first claim, suppose $x \in X$ is a T-supported sequence for some $T > T^*$. Denote by s_j the agent that makes the j^{th} support (recall that in Lemma A.2 h_{T-1} is the agent that has the $(T-1)^{\text{th}}$ positive signal; here s_j is not necessarily the agent observing the j^{th} positive signal). There are two possibilities. If s_{T^*} is part of an UP cascade, that is, if she supports regardless of her private signals, then the formation of the UP cascade is not affected by reducing the AoN target to T^*. Therefore, x is also T^*-supported.

If s_{T^*} is not part of an UP cascade, then all of the first T^* supporters have a positive private signal. If the AoN target is reduced to T^*, it does not affect the decision of all agents $i \leq s_{T^*-1}$. Agent s_{T^*}, as the gatekeeper, supports only if the number of positive signals exceeds the number of negative signals by at least k^*, that is, $\sum_{i=1}^{s_{T^*}} x_i \geq k^*$. This is the case because

$$\sum_{i=1}^{s_{T^*}} x_i = T^* + (T^* - s_{T^*}) = 2T^* - s_{T^*} \geq 2T^* - N + (T - T^*) \geq 2T^* - N + 1 \geq k^*.$$

The first inequality comes from the fact that $s_{T^*} + (T - T^*) \leq s_T \leq N$. Therefore, x is T^*-supported too. Moreover, notice that if $s_{T^*} + (T - T^*) = s_T$, then agent s_{T^*+1} must observe a positive signal and support, starting an UP cascade conditional on T^*. If $s_{T^*} + (T - T^*) < s_T$, we have

$$\sum_{i=1}^{s_{T^*}} x_i = T^* + (T^* - s_{T^*}) = 2T^* - s_{T^*} \geq 2T^* - N + (T - T^*) + 1 \geq 2T^* - N + 2 \geq k^* + 1.$$

Again, we then have that s_{T^*} is part of an UP cascade for T^*.

In the last step, it is easy to check that the following sequence is T^*-supported and not T-supported for any $T > T^*$: $(\underbrace{-1, \ldots, -1}_{\frac{N-k^*}{2}}, \underbrace{1, \ldots, 1}_{\frac{N+k^*}{2}})$ when $N + k^*$ is even, and $(\underbrace{-1, \ldots, -1}_{\frac{N-k^*-1}{2}}, \underbrace{1, \ldots, 1}_{\frac{N+k^*-1}{2}}, -1)$ when $N + k^*$ is odd.

Step 3: The Proof of $\pi(p^*, T) < \pi(p^*, T^*)$, for $T < T^*$ and $k^* > 0$

Notice that given Proposition 1 and $k^* > 0$, if $T < k^*$ then the project will not be implemented for sure. Thus, we only consider the case in which $k^* \leq T$. We first show that given the price p^*, the probability of reaching the AoN target ($\Pr(A_N \geq T | p^*, T)$) strictly increases with T for $k^* \leq T \leq T^*$. We then show that the expected number of supporters conditional on reaching the AoN target ($\mathbb{E}[A_N | A_N \geq T, T]$) is also increasing in T. These two results are enough to conclude that the proposer's expected profit is increasing in T for $k^* \leq T \leq T^*$.

First, we show that $\Pr(A_N \geq T | p^*, T) \geq \Pr(A_N \geq T - 1 | p^*, T - 1)$, for $T \leq T^*$. Recall that $\tilde{X}_{T-1/T}$ is the set of all sequences of signals that are $(T-1)$-supported but not T-supported, and $\tilde{X}_{T/T-1}$ is the set of sequences of signals that are T-supported but not $(T-1)$-supported. We then need only to show that $\Pr(\tilde{X}_{T/T-1}) \geq \Pr(\tilde{X}_{T-1/T})$. We first prove a useful lemma here. Let $\varphi_{k+1,i}$ denote the probability that an UP cascade starts after agent i. Before reaching the threshold, because agents are truth telling before an UP cascade starts, the arrival of an UP cascade is equivalent to the first passage time of a one-dimensional biased random walk. Using results on hitting times from Van der Hofstad and Keane (2008), we can then compute $\varphi_{k+1,i}$.

LEMMA A.4: *Suppose $\bar{k}(p) = k$ (recall that $\bar{k}(p)$ is defined just before Proposition 1). We then have:*

(a) The probability that an UP cascade starts after agent i is

$$\varphi_{k+1,i} = \frac{k+1}{i}\binom{i}{\frac{i+k+1}{2}}[q(1-q)]^{\frac{i-k-1}{2}}\frac{(1-q)^{k+1}+q^{k+1}}{2}, \tag{A.10}$$

where

$$\binom{i}{\frac{i+j}{2}} = \begin{cases} \frac{i!}{\frac{i+j}{2}!\frac{i-j}{2}!} & \text{if } i \geq j \text{ and } j+i \text{ even}, \\ 0 & \text{otherwise.} \end{cases} \tag{A.11}$$

(b) For a given price and threshold pair (p, T), the probability of reaching the threshold at agent i without a prior UP cascade is $\frac{q^k+(1-q)^k}{q^{k+1}+(1-q)^{k+1}}\varphi_{k+1,i+1}$.

PROOF: First, we restate a standard result of the hitting-time theorem (Van der Hofstad and Keane (2008)).

LEMMA A.5 (Hitting Time Theorem): *Fix $n \geq 1$. Let $\{Y_i\}_{i=1}^{\infty}$ be a sequence of independently and identically distributed random variables Y_i taking values in $\{\cdots, -2, -1, 0, 1\}$. Define $S_n = \sum_{i=0}^{n} Y_i$, where $Y_0 = 0$. Define the stopping (hitting) time $\tau_k = \inf\{m \geq 0 : S_m = k\}$. Then,*

$$\Pr(\tau_k = n) = \frac{k}{n}\Pr(S_n = k). \tag{A.12}$$

We are now ready to prove Lemma A.4.

Part (a). Note that an UP cascade starts after agent i if and only if $\tau_{k+1} = i$, once we replace Y_i in Lemma A.5 with X_i in our setting. Moreover, we can directly derive $\Pr(S_i = k+1|V \in \{0, 1\})$ by combinatorial calculation as

$$\Pr(\tau_{k+1} = i|V = 1) = \frac{k+1}{i}\Pr(S_i = k+1|V = 1) = \frac{k+1}{i}\binom{i}{\frac{i+k+1}{2}}q^{\frac{i+k+1}{2}}(1-q)^{\frac{i-k-1}{2}},$$

and

$$\Pr(\tau_{k+1} = i|V = 0) = \frac{k+1}{i}\Pr(S_i = k+1|V = 0) = \frac{k+1}{i}\binom{i}{\frac{i+k+1}{2}}(1-q)^{\frac{i+k+1}{2}}q^{\frac{i-k-1}{2}}.$$

Moreover, note that

$$\varphi_{k+1,i} = \Pr(\tau_{k+1} = i) = \sum_{j\in\{0,1\}}\Pr(V = j)\Pr(\tau_{k+1} = i|V = j).$$

The proof concludes with simple algebra.

Part (b). Denote

$$A = \{\mathcal{H}_i : \text{Reaching the threshold at } i \text{ without prior UP cascade}\}.$$

Obviously, $k(\mathcal{H}_i) = \bar{k}(p)$. Thus, if $x_{i+1} = 1$, then we have an UP cascade starting at $i+1$, or $\tau_{k+1} = i+1$. We therefore have

$$\Pr(A|V=j) = \frac{\Pr(\tau_{k+1} = i+1|V=j)}{\Pr(x_{i+1} = 1|V=j)}$$

for $j \in \{0, 1\}$. This further implies

$$\begin{aligned}\Pr(A) &= \sum_{j\in\{0,1\}} \Pr(V=j)\Pr(A|V=j)\\ &= \sum_{j\in\{0,1\}} \Pr(V=j)\frac{\Pr(\tau_{k+1}=i+1|V=j)}{\Pr(x_{i+1}=1|V=j)}.\end{aligned}$$

Plugging in the expressions for both $\Pr(\tau_{k+1} = i+1|V=j)$ (see the proof of Part (a) above) and $\Pr(x_{i+1} = 1|V=j)$ concludes the proof. □

Returning to the main proof for Step 3. Following the proof of Lemma A.3, for every sequence of signals $x = (\underbrace{\ldots}_{h_{T-1}-1}, 1, -1, -1, \ldots) \in \tilde{X}_{T-1/T}$, there exists a corresponding sequence $x'(x) = (\underbrace{\ldots}_{h_{T-1}-1}, -1, 1, 1, \ldots) \in \tilde{X}_{T/T-1}$ in which only the three middle signals are reversed. The function $x'(x)$ is an injective function, and thus,

$$\begin{aligned}&\Pr(\tilde{X}_{T/T-1}) = \frac{1}{2}\left[\Pr(\tilde{X}_{T/T-1}|V=1) + \Pr(\tilde{X}_{T/T-1}|V=0)\right]\\ &\geq \frac{1}{2}\left[\frac{q}{1-q}\Pr(\tilde{X}_{T-1/T}|V=1) + \frac{1-q}{q}\Pr(\tilde{X}_{T-1/T}|V=0)\right]\\ &\qquad\Rightarrow \Pr(\tilde{X}_{T/T-1}) - \Pr(\tilde{X}_{T-1/T}) \geq\\ &\frac{2q-1}{2}\left[\frac{1}{1-q}\Pr(\tilde{X}_{T-1/T}|V=1) - \frac{1}{q}\Pr(\tilde{X}_{T-1/T}|V=0)\right].\end{aligned}$$

The first inequality comes from the fact that $x'(x)$ is an injection but not necessarily a bijection. Consequently, we need only to show

$$\frac{\Pr(\tilde{X}_{T-1/T}|V=1)}{\Pr(\tilde{X}_{T-1/T}|V=0)} \geq \frac{1-q}{q}.$$

To see this, note that

$$\mathbb{E}\big[V|x \in \tilde{X}_{T-1/T}\big] = V_{k^*-2}$$

$$\Rightarrow \frac{\Pr(\tilde{X}_{T-1/T}|V=1)}{\Pr(\tilde{X}_{T-1/T}|V=0)} = \frac{V_{k^*-2}}{1-V_{k^*-2}} \geq \frac{V_{-1}}{1-V_{-1}} = \frac{1-q}{q},$$

where the first equality comes from the fact that all sequences in $\tilde{X}_{T-1/T}$ are of the form $(\underbrace{\ldots}_{h_{T-1}-1}, 1, -1, -1, \ldots)$ (see the proof of Lemma A.3). We therefore have $\Pr(\tilde{X}_{T/T-1}) \geq \Pr(\tilde{X}_{T-1/T})$ for $T \leq T^*$ and $k^* > 0$.

We next discuss the number of supporters conditional on reaching the AoN target. Based on the proof of Lemma A.3, we know that for each $x \in X$ that is $(T-1)$-supported, there exists a distinct sequence $x' \in X$ that is T-supported and has at least the same number of supporting agents. Moreover, for $x \in \tilde{X}_{T-1/T}$, the corresponding $x'(x)$ has a strictly higher number of supporters.

As a result, the probability of reaching the AoN target is greater for T than $T-1$, and the expected number of supporters conditional on reaching the target is strictly greater for T than $T-1$. Therefore, for any $\nu < V_N$, $\pi(p^*, T) > \pi(p^*, T-1)$ for $k^* \leq T \leq T^*$. By combining the results of Steps 2 and 3, we conclude that when the equilibrium price $p^* = V_{k^*}$ satisfies $k^* > 0$, $\pi(p^*, T^*) > \pi(p^*, T)$ for any $T \neq T^*$, so the optimal AoN target is $T^\star = \lfloor \frac{N+k^\star}{2} \rfloor$ when $k^* > 0$.

Step 4: Optimality of T^* when $k^* = 0$, $q(1-q) > \frac{1}{6}$

For $k^* = 0$ ($p^* = \frac{1}{2}$), the proof of Step 2 also applies here, so $\pi(\frac{1}{2}, T) < \pi(\frac{1}{2}, T^*)$, for $T > T^*$. Therefore, we need only to show $\pi(\frac{1}{2}, T) < \pi(\frac{1}{2}, T^*)$ for $T < T^*$. To be more specific, we show that any strategy $(p = \frac{1}{2}, T-1)$, $2 \leq T \leq T^*$, is dominated by $(\frac{1}{2}, T)$ (notice that $T^* = 1$ is effectively a no-AoN target).

Given $p = \frac{1}{2}$, for any signal sequence x that has agent $2T$ as part of an UP cascade, the first $2T$ agents must observe at least $T+1$ positive signals, the signal sequence is both $(T-1)$-supported and T-supported, and the number of supporters is the same for $T-1$ and T. So the proposer is indifferent between $(\frac{1}{2}, T-1)$ and $(\frac{1}{2}, T)$ when agent $2T$ is part of an UP cascade.

Define $\mathbb{Q}_m = \{x | \sum_{i=1}^{j} x_i \leq 0, \forall j \leq m, \sum_{i=1}^{m} x_i = 0\}$. For any $x \in \mathbb{Q}_m$, agent m is not part of an UP cascade. Then, the sets $\tilde{X}_{T-1/T}$ and $\tilde{X}_{T/T-1}$ can be characterized as

$$\tilde{X}_{T-1/T} = \{x | x \in \mathbb{Q}_{2T-2}, x_{2T-1} = x_{2T} = -1\},$$

$$\tilde{X}_{T/T-1} = \mathbb{Q}_{2T} / \{x | x \in \mathbb{Q}_{2T-2}, x_{2T-1} = -1, x_{2T} = 1\}.$$

Notice that there is no need here to consider $x_{2T-1} = 1, x_{2T} = -1$ because this would create an UP cascade. The first result follows directly from the proof of

Lemma A.3, which suggests why $\sum_{i=1}^{2T-2} x_i = 0$ for any $x \in \tilde{X}_{T-1/T}$. Lemma A.4 gives the probability of $\mathbb{Q}_m$,

$$\begin{aligned}\Pr(\mathbb{Q}_m) &= \frac{1}{2}\Pr(\mathbb{Q}_m|V=1) + \frac{1}{2}\Pr(\mathbb{Q}_m|V=0)\\ &= \frac{1}{2}\frac{\Pr\left(\mathbb{H}^U_{m+1}|V=1\right)}{q} + \frac{1}{2}\frac{\Pr\left(\mathbb{H}^U_{m+1}|V=0\right)}{1-q},\end{aligned}$$

where $\mathbb{H}^U_m$ is the set of histories that an UP cascade starts after agent m. Let $\pi(p,T,\mathbb{Z})$ be the expected revenue on event set $\mathbb{Z}$, given strategy (p,T). Then,

$$\begin{aligned}\frac{\pi(\frac{1}{2},T,\tilde{X}_{T/T-1})}{\pi(\frac{1}{2},T-1,\tilde{X}_{T-1/T})} &\geq \frac{\Pr(\tilde{X}_{T/T-1})}{\Pr(\tilde{X}_{T-1/T})}\frac{T}{T-1}\\ &= \frac{\Pr(\mathbb{Q}_{2T}) - \Pr(\mathbb{Q}_{2T-2})q(1-q)}{\Pr(\mathbb{Q}_{2T-2})\frac{(1-q)^2+q^2}{2}}\frac{T}{T-1}\\ &= \frac{\frac{1}{2}\frac{\Pr(\mathbb{H}^U_{2T+1}|V=1)}{q} + \frac{1}{2}\frac{\Pr(\mathbb{H}^U_{2T+1}|V=0)}{1-q} - \frac{1}{2}\frac{\Pr(\mathbb{H}^U_{2T-1}|V=1)q(1-q)}{q} - \frac{1}{2}\frac{\Pr(\mathbb{H}^U_{2T-1}|V=0)q(1-q)}{1-q}}{[\frac{1}{2}\frac{\Pr(\mathbb{H}^U_{2T-1}|V=1)q^2}{1-q} + \frac{1}{2}\frac{\Pr(\mathbb{H}^U_{2T-1}|V=0)(1-q)^2}{q}]\frac{T-1}{T}}\\ &= \frac{6T}{T+1}\frac{q(1-q)}{(1-q)^2+q^2} \geq \frac{4q(1-q)}{1-2q(1-q)},\end{aligned}$$

where the first inequality comes from the fact that any $x \in \tilde{X}_{T-1/T}$ has exactly $T-1$ supporting agents while any $x \in \tilde{X}_{T/T-1}$ has at least T supporters, as we show in Lemma A.3. The last inequality applies the fact that $T \geq 2$. When $q(1-q) > \frac{1}{6}$, $\pi(\frac{1}{2},T,\tilde{X}_{T/T-1}) > \pi(\frac{1}{2},T-1,\tilde{X}_{T-1/T})$ and thus $(\frac{1}{2},T-1)$ is dominated by $(\frac{1}{2},T)$.

Step 5: Optimality of T^* when $k^*=0$, $q(1-q) \leq \frac{1}{6}$

Similar to Step 4, we need only to show that for any $2 \leq T \leq T^*$, $(\frac{1}{2},T-1)$ is a dominated strategy. To be more specific, we show that $(\frac{1}{2},T-1)$ is dominated by (q,T). To do so, we decompose all possible implementation histories under strategy $(\frac{1}{2},T-1)$ into several sets and show that, for each set, a corresponding distinct set of implementation histories under strategy (q,T) is associated with more profits.

When $(1-q)q \leq \frac{1}{6}$, we have $q \geq \frac{1}{2} + \frac{\sqrt{3}}{6} > \frac{3}{4}$. For $p^* = \frac{1}{2}$ and AoN threshold $T-1$, the project would be implemented when there is already an UP cascade by agent $2T-2$ or when there is no UP cascade by agent $2T-2$ and the $2T-2^{\text{th}}$ agent is the $T-1^{\text{th}}$ supporting agent. This is because if there is no UP cascade by agent $2T-2$ and she is not the $T-1^{\text{th}}$ supporting agent, then there are weakly more than T rejecting agents and weakly less than $T-2$ supporting agents among the first $2T-2$ agents, and therefore when we reach the $T-1^{\text{th}}$ agent with positive signal, it has to be the case that $k<0$ at that point (and therefore that agent would not support the project). It suffices to show that in each scenario, the alternative strategy is better for the proposer.

(1) When there is already an UP cascade by agent $2T-2$, let $\mathbb{H}_i^U$ be the set of histories that result in an UP cascade starting after agent $i \leq 2T-2$. Given any $\mathcal{H}_i \in \mathbb{H}_i^U$, $\mathbb{E}[V|k(\mathcal{H}_i)] = q$, and denote the number of supporters by $A_N(\mathbb{H}_i^U)$. If $x_{i+1} = 1$, then there would be an UP cascade starting after agent $i+1$ for the strategy $(p^* = q, T)$, and the number of supporting agents is $A_N(\mathbb{H}_i^U)$. Let $\pi(p, T|\ \mathbb{H}_i^U)$ be the expected payoffs for the proposer conditional on strategy (p, T) and event $\mathbb{H}_i^U$. We have that $(p^* = q, T)$ dominates $(\frac{1}{2}, T-1)$ conditional on $\mathbb{H}_i^U$ because

$$\begin{aligned}
\pi\left(q, T|\ \mathbb{H}_i^U\right) &\geq (q-\nu)A_N\left(\mathbb{H}_i^U\right)\left[\Pr\left(V=1|\mathbb{H}_i^U\right)q + \Pr\left(V=0|\mathbb{H}_i^U\right)(1-q)\right] \\
&= A_N\left(\mathbb{H}_i^U\right)(q-\nu)[(1-q)^2+q^2] \\
&> A_N\left(\mathbb{H}_i^U\right) \times \left(\frac{3}{4}-\nu\right) \times (1-2(1-q)q) \\
&\geq A_N\left(\mathbb{H}_i^U\right) \times \left(\frac{3}{4}-\nu\right) \times \frac{2}{3} \\
&\geq \left(\frac{1}{2}-\nu\right) A_N\left(\mathbb{H}_i^U\right) \\
&= \pi\left(\frac{1}{2}, T-1|\ \mathbb{H}_i^U\right).
\end{aligned}$$

The second inequality comes from the fact that $q(1-q) \leq \frac{1}{6}$.

(2) When there is no UP cascade by agent $2T-2$ but the $(2T-2)^{\text{th}}$ agent is the $(T-1)^{\text{th}}$ supporting agent, $x \in \mathbb{Q}_{2T-2}$. Consider the following two sets of histories for strategy (q, T):
   (a) $\mathbb{Q}_{2T-1}^A = \{x | x \in \mathbb{Q}_{2T-2}, x_{2T-1} = 1\}$:
   Obviously the threshold T is met. Since given any $\mathcal{H}_{2T-2} \in \mathbb{Q}_{2T-2}$, there are an equal number of positive and negative signals by agent $2T-2$, we have

$$\Pr\left(\mathbb{Q}_{2T-1}^A\right) = \Pr(\mathbb{Q}_{2T-2})[\Pr(V=1|\mathbb{Q}_{2T-2})q + \Pr(V=0|\mathbb{Q}_{2T-2})(1-q)] = \frac{1}{2}\Pr(\mathbb{Q}_{2T-2}).$$

   We next discuss the expected number of supporting agents:
   For event $\mathbb{Q}_{2T-2}^A$ under strategy $(\frac{1}{2}, T-1)$, the UP cascade starts after agent $2T-1$ and the number of supporting agents is $N-T+1$, the maximum conditional on $\mathbb{Q}_{2T-2}$. For event $\mathbb{Q}_{2T-1}^A$ under strategy (q, T), if $x_{2T} = 1$, the UP cascade starts after agent $2T$ and the number of supporting agents is $N-T+1$. The associated conditional

probability is

$$\Pr\left(x_{2T}=1|\mathbb{Q}^{A}_{2T-1}\right)=q^2+(1-q)^2\geq\frac{2}{3}>\frac{1}{2}.$$

For event $\mathbb{Q}_{2T-2}$ under strategy $(\frac{1}{2},T-1)$,

$$Pr(x_{2T-1}=-1|\mathbb{Q}_{2T-2})=\frac{1}{2}.$$

In contrast, for event $\mathbb{Q}^{A}_{2T-1}$, under strategy (q,T), $Pr(x_{2T}=-1|\mathbb{Q}^{A}_{2T-1})=2q(1-q)<\frac{1}{2}$. For each possible subsequence $z=\{x_{2T},x_{2T+1},\ldots,x_N\}$ of sequence $x\in\mathbb{Q}_{2T-2}$, and for $x_{2T-1}=-1$, let $A_N(\frac{1}{2},T-1|\mathbb{Q}_{2T-2},x_{2T-1}=-1,z)$ be the associated number of supporting agents under strategy $(\frac{1}{2},T-1)$. The corresponding subsequence $z'=\{x_{2T},x_{2T+1},\ldots,x_{N-1}\}$ satisfies

$$A_N\left(q,T|\mathbb{Q}^{A}_{2T-1},x_{2T}=-1,z'\right)>A_N\left(\frac{1}{2},T-1|\mathbb{Q}_{2T-2},x_{2T-1}=-1,z\right).$$

This inequality holds because in each scenario, just before the corresponding subsequence $z(z')$ starts, the posterior is $\bar{k}(p)-1$, the project would be implemented for sure, and there are T supporters in $\mathbb{Q}^{A}_{2T-1}$ before z' but only $T-1$ supporting decisions in $\mathbb{Q}_{2T-2}$ before z. So

$$\mathbb{E}\left[A_N(q,T)|\mathbb{Q}^{A}_{2T-1},x_{2T}=-1\right]\geq\mathbb{E}\left[A_N\left(\frac{1}{2},T-1\right)|\mathbb{Q}_{2T-2},x_{2T-1}=-1\right].$$

Then,

$$\begin{aligned}
&\mathbb{E}\Big[A_N(q,T)|\mathbb{Q}^{A}_{2T-1}\Big]\\
&=\sum_{-1,1}\Pr(x_{2T}=i|\mathbb{Q}^{A}_{2T-1})\mathbb{E}\Big[A_N(q,T)|\mathbb{Q}^{A}_{2T-1},x_{2T}=i\Big]\\
&=(q^2+(1-q)^2)(N-T+1)+\\
&\quad 2q(1-q)\mathbb{E}\Big[A_N(q,T)|\mathbb{Q}^{A}_{2T-1},x_{2T}=-1\Big]\\
&>\frac{1}{2}(N-T+1)+\frac{1}{2}\mathbb{E}\left[A_N(q,T)|\mathbb{Q}^{A}_{2T-1},x_{2T}=-1\right]\\
&>\frac{1}{2}(N-T+1)+\frac{1}{2}\mathbb{E}\left[A_N\left(\frac{1}{2},T-1\right)|\mathbb{Q}_{2T-2},x_{2T-1}=-1\right]\\
&=\sum_{-1,1}\Pr(x_{2T-1}=i|\mathbb{Q}_{2T-2})\mathbb{E}\left[A_N\left(\frac{1}{2},T-1\right)|\mathbb{Q}_{2T-2},x_{2T-1}=i\right]\\
&=\mathbb{E}\left[A_N\left(\frac{1}{2},T-1\right)|\mathbb{Q}_{2T-2}\right],
\end{aligned}$$

where the first inequality comes from the fact that $N - T - 1$ is the maximum number of possible supporters conditional on $\mathbb{Q}_{2T-2}$.

(b) Consider the set $\mathbb{Q}^B_{2T-1} = \{x| \sum_{i=1}^{j} x_i \leq 1, \forall 1 \leq j \leq 2T-1, \sum_{i=1}^{2T-3} x_i = 1, x_{2T-2} = -1, x_{2T-1} = 1\}$:
The event $\mathbb{Q}^B_{2T-1}$ is such that there is no UP cascade (with respect to $p^* = q$) by agent $2T-2$, and $k(\mathcal{H}_{2T-3}) = 1$, $x_{2T-2} = -1$, and $x_{2T-1} = 1$. Obviously the threshold T is met. Notice that for strategy (q, T), histories in this set are distinct from those we discuss above (in case 1 we only cover UP cascades for strategy (q, T)). For any sequence $x \in \mathbb{Q}^B_{2T-1}$, there is a mapping $x^A(x) = \{x_{2T-2}, x_1, x_2, \ldots, x_{2T-3}, x_{2T-1}, \ldots\}$. The mapping $x^A(x)$ is a bijection that establishes a one-to-one mapping between finite sets $\mathbb{Q}^B_{2T-1}$ and $\mathbb{Q}^A_{2T-1}$. Following the discussion in part (a), we have $\Pr(\mathbb{Q}^B_{2T-1}) = \Pr(\mathbb{Q}^A_{2T-1}) = \frac{1}{2}\Pr(\mathbb{Q}_{2T-2})$, and the expected number of supporters conditional on event $\mathbb{Q}^B_{2T-1}$ and strategy (q, T) is higher than that of event $\mathbb{Q}_{2T-2}$ and strategy $(\frac{1}{2}, T-1)$.

Since $\Pr(\mathbb{Q}^B_{2T-1}) + \Pr(\mathbb{Q}^A_{2T-1}) = \Pr(\mathbb{Q}_{2T-2})$, and in either case there are more supporting agents who generate higher profit $q - \nu > \frac{1}{2} - \nu$, So $(p^* = q, T)$ dominates $(\frac{1}{2}, T-1)$ when there is no cascade and $(1-q)q \leq \frac{1}{6}$.

Step 6: Optimality of T^* when $k^* = -1$

Note that for $k^* = -1$ ($p^* = 1 - q$), an UP cascade is reached from the first agent. Therefore, regardless of the choice of T, all agents support the proposal. Therefore, $T = T^*$ is an optimal choice.

In conclusion, Steps 2 to 6 show that T^* is the proposer's weakly dominant strategy, and it is a strictly dominant strategy when T different choices may lead to different equilibrium expected proceeds. □

D. Proof of Corollary 1

PROOF: Recall that s_j is the j^{th} supporting agent. We show that there is no DOWN cascade unless the following holds simultaneously:

(1) $N + k^*$ is odd.
(2) There is no UP cascade.
(3) $N - 3 \leq s_{T^*-1} \leq N - 1$.
(4) $x_j = -1, \forall \quad s_{T^*-1} < j \leq N - 1$.

First, if there is an UP cascade, then there would be no DOWN cascade. Second, if there are fewer than $T^* - 1$ supporting agents, then from Proposition 1 there would be no DOWN cascade.

The only remaining case is when there is no UP cascade by agent s_{T^*-1}. Since there is no UP cascade yet, then by the construction of $T^* = \lfloor \frac{N+k^*}{2} \rfloor$, $s_{T^*-1} \geq$

$2(T^* - 1) - k^* \geq N - 3$ (as otherwise there would be strictly fewer than $T^* - 1 - k^*$ rejecting agents when we reach s_{T^*-1}). To be more specific:

(1) If $N + k^*$ is even, then $s_{T^*-1} \geq N - 2$. When $s_{T^*-1} = N$, then from Proposition 1 there would be no DOWN cascade. When $s_{T^*-1} \in \{N-2, N-1\}$, $k_{s_{T^*-1}} = k^* + N - 2 - s_{T^*-1}$ and there would be no DOWN cascade (a DOWN cascade starts after $k = k^* - 2$).

(2) If $N + k^*$ is odd, then $s_{T^*-1} \geq N - 3$. When $s_{T^*-1} = N$, then from Proposition 1 there would be no DOWN cascade. When $s_{T^*-1} \in \{N-3, N-2, N-1\}$, $k_{s_{T^*-1}} = k^* + N - 3 - s_{T^*-1}$ and there exists a DOWN cascade after agent $N-1$ if all agents $s_{T^*-1} < j \leq N-1$ observe negative signals.

□

E. *Proof of Proposition 3*

PROOF: When the price is $p = V_{-1} = 1 - q$, the proposer's expected profit is $(1 - q - \nu)N$. When the price is $p = V_k$, $k \in \{0, 1, \ldots, N\}$, based on Lemma A.4, the proposer's expected profit given the corresponding optimal AoN target $T(N, k) = \lfloor \frac{N+k}{2} \rfloor$ is

$$\pi\left(V_k, \left\lfloor \frac{N+k}{2} \right\rfloor\right) = \begin{cases} (V_k - \nu)\left[\sum_i^N \varphi_{k+1,i}(N - \frac{i-k-1}{2}) + \frac{(1-q)^k q + (1-q)q^k}{(1-q)^{k+1} + q^{k+1}} \varphi_{k+1,N} \frac{N+k-1}{2}\right] & \text{if } k + N \text{ odd,} \\ (V_k - \nu)\left[\sum_i^{N-1} \varphi_{k+1,i}(N - \frac{i-k-1}{2}) + \frac{(1-q)^{k+1} + q^{k+1}}{(1-q)^{k+2} + q^{k+2}} \varphi_{k+1,N+1} \frac{N+k}{2}\right] & \text{if } k + N \text{ even.} \end{cases} \tag{A.13}$$

We start by proving the following lemma.

LEMMA A.6: *Let $\bar{k}(\nu) \in \{0, 1, 2, \ldots\}$ be the smallest integer satisfying $V_{\bar{k}(\nu)} \geq \nu$. For each $k \in \{\bar{k}(\nu), \bar{k}(\nu) + 1, \bar{k}(\nu) + 2, \ldots\}$, there exists a finite positive integer $\underline{N}(k)$ such that for $\forall\, N \geq \underline{N}(k)$, $\pi(V_k, \lfloor \frac{N+k}{2} \rfloor) > \pi(V_{k-1}, \lfloor \frac{N+k-1}{2} \rfloor)$, where the arguments in π are the price and AoN threshold, respectively.*

PROOF: To show the existence of $\underline{N}(k)$, we first prove the existence of $\underline{N}(0)$. We then proceed to the $k \geq 1$ case. From the standard Gambler's Ruin problem, we know that as $N \to \infty$, for a given V_k, the conditional probability that an UP cascade occurs at a finite time is one if $V = 1$ and $\frac{(1-q)^{k+1}}{q^{k+1}}$ if $V = 0$ (Feller (1968), p. 347, equation (2.8)).

Note that the total profit when $p = V_{-1}$ is $(1 - q - \nu)N$. Furthermore, for $p = V_0 = \frac{1}{2}$, an UP cascade starts if $\sum_{j=1}^{i} x_j = 1$ for some $1 \leq j < 2T$, where

$T = \lfloor \frac{N}{2} \rfloor$. Because $(1-q)q < \frac{1}{4}$, we have

$$\begin{aligned}\lim_{N\to\infty} \frac{\pi(V_0, \lfloor\frac{N}{2}\rfloor)}{N} &= (V_0 - \nu)\left(\Pr(V=1) + \Pr(V=0)\frac{1-q}{q}\right)\\ &= \left(\frac{1}{2} - \nu\right)\left(\frac{1}{2} + \frac{1-q}{2q}\right) > \left(\frac{1}{2} - \nu\right)2(1-q)\\ &> 1 - q - \nu = V_{-1} - \nu.\end{aligned}$$

Since $\varphi_{1,i}$ is strictly positive, there exists a strictly positive integer $N_1(0)$ such that

$$(V_0 - \nu)\sum_{i=1}^{N_1(0)} \varphi_{1,i} > 1 - q - \nu.$$

Let $D_0 = (V_0 - \nu)\sum_{i=1}^{N_1(0)} \varphi_{1,i} - (1 - q - \nu) > 0$, $Q_0 = (V_0 - \nu)\sum_{i=1}^{N_1(0)} \varphi_{1,i}\frac{i-1}{2}$, and let $\underline{N}(0)$ be the smallest integer such that $\underline{N}(0) > \max\{N_1(0), \frac{Q_0}{D_0}\}$. Then for any $N \geq \underline{N}(0)$,

$$\begin{aligned}\pi(V_0, \lfloor\frac{N}{2}\rfloor) &\geq (V_0 - \nu)\sum_{i=1}^{\underline{N}(0)} \varphi_{1,i}(N - \frac{i-1}{2}) \geq N(V_0 - \nu)\sum_{i=1}^{N_1(0)} \varphi_{1,i} - Q_0\\ &> \frac{Q_0}{D_0}D_0 + (1 - q - \nu)N - Q_0 = (1 - q - \nu)N.\end{aligned}$$

Next, we prove the existence of $\underline{N}(k)$ for $k > 0$. For each $k \geq 1$, and the time i UP cascade arrival rate $\varphi_{k+1,i+1}$, there exists a corresponding $\varphi_{k,i}$ for price V_{k-1}, and we have

$$\begin{aligned}\frac{(V_k - \nu)\varphi_{k+1,i+1}}{(V_{k-1} - \nu)\varphi_{k,i}} &\geq \frac{V_k\varphi_{k+1,i+1}}{V_{k-1}\varphi_{k,i}}\\ &= \frac{V_k \frac{k+1}{i+1}\frac{(i+1)!}{\frac{i+k+2}{2}!\,\frac{i-k}{2}!}[(1-q)q]^{\frac{i-k}{2}}\frac{(1-q)^{k+1}+q^{k+1}}{2}}{V_{k-1}\frac{k}{i}\frac{i!}{\frac{i+k}{2}!\,\frac{i-k}{2}!}[(1-q)q]^{\frac{i-k}{2}}\frac{(1-q)^k+q^k}{2}}\\ &= q\frac{k+1}{k}\frac{i}{\frac{i+k}{2}+1}\left(1 + \frac{[q(1-q)]^{k-1}(q-(1-q))^2}{((1-q)^k + q^k)^2}\right).\end{aligned}$$

Because $\lim_{i\to\infty} q\frac{i}{\frac{i+k}{2}+1} = 2q > 1$, for each k, the ratio $\frac{V_k\varphi_{k+1,i+1}}{V_{k-1}\varphi_{k,i}}$ is monotonically increasing in i for large enough values of i, and thus, there exists an integer

N_1 that $\frac{V_k\varphi_{k+1,i+1}}{V_{k-1}\varphi_{k,i}} \geq 1$ whenever $i \geq N_1$. We then have

$$\lim_{N\to\infty}(V_k-\nu)\sum_{i=1}^{N-1}\varphi_{k+1,i+1} = (V_k-\nu)\left(\frac{1}{2}+\frac{\frac{(1-q)^{k+1}}{q^{k+1}}}{2}\right)$$
$$= \frac{V_k-\nu}{V_k}\frac{1}{2}\frac{q^k}{q^k+(1-q)^k}\frac{(1-q)^{k+1}+q^{k+1}}{q^{k+1}} = \frac{V_k-\nu}{V_k}\frac{1}{2q}\frac{(1-q)^{k+1}+q^{k+1}}{(1-q)^k+q^k}$$
$$> \frac{V_k-\nu}{V_k}\frac{1}{2q}\frac{(1-q)^k+q^k}{(1-q)^{k-1}+q^{k-1}} = \frac{V_k-\nu}{V_k}V_{k-1}\left(\frac{1}{2}+\frac{\frac{(1-q)^k}{q^k}}{2}\right)$$
$$\geq (V_{k-1}-\nu)\left(\frac{1}{2}+\frac{\frac{(1-q)^k}{q^k}}{2}\right) = \lim_{N\to\infty}(V_{k-1}-\nu)\sum_{i=1}^{N}\varphi_{k,i},$$

where we use the Cauchy–Schwarz inequality to derive $(q^{k+1}+(1-q)^{k+1})(q^{k-1}+(1-q)^{k-1}) > (q^k+(1-q)^k)^2$.

Given that $\lim_{N\to\infty}\sum_{i=1}^{N}\varphi_{k+1,i+1}$ converges to a finite number, there exists an integer $N_2 \geq N_1$ such that

$$D \equiv (V_k-\nu)\sum_{i=1}^{N_2-1}\varphi_{k+1,i+1} - (V_{k-1}-\nu)\sup_{N\geq N_2}\left\{\sum_{i=1}^{N}\varphi_{k,i}+\frac{q^{k-1}+(1-q)^{k-1}}{(1-q)^k+q^k}\varphi_{k,N+1}\right\} > 0,$$

where $\frac{q^{k-1}+(1-q)^{k-1}}{(1-q)^k+q^k}\varphi_{k,N+1}$ is the probability that there is no UP cascade and agent N is the T^{th} supporting agent given price V_{k-1}. Let $Q \equiv (V_k-\nu)\sum_{i=1}^{N_2-1}\varphi_{k+1,i+1}\frac{i-k}{2}$. Then, for each k, let $\underline{N}(k)$ be the smallest integer that is larger than $\max\{N_2, \frac{Q}{D}\}$. Then, for any $N \geq \underline{N}(k)$,

$$\pi\left(V_k, \left\lfloor\frac{N+k}{2}\right\rfloor\right) - \pi\left(V_{k-1}, \left\lfloor\frac{N+k-1}{2}\right\rfloor\right) > ND - Q > \underline{N}(k)D - Q \geq 0.$$

□

Given Lemma A.6, for $\forall N$ and the corresponding optimal price V_k, one can construct a finite agent base $\overline{N} \equiv \max\{\underline{N}(\bar{k}(\nu)), \underline{N}(\bar{k}(\nu)+1), \underline{N}(\bar{k}(\nu)+2)\ldots\underline{N}(k+1)\}$ such that for $\forall N \geq \overline{N}$, we have

$$\begin{aligned}\pi(V_{k+1}, T(V_{k+1}))&>\pi(V_k, T(V_k)) > \pi(V_{k-1}, T(V_{k-1}))\\ &>\pi(V_{k-2}, T(V_{k-2})) > \ldots > \pi(V_{\bar{k}(\nu)}, T(V_{\bar{k}(\nu)})).\end{aligned} \tag{A.14}$$

That is, for $\forall N \geq \overline{N}$, the optimal price $p^* \geq V_{k+1} > V_k$. Moreover, this result implies that for $\forall k \in \mathbb{Z}$, there exists a finite number $\overline{N}$ such that for $\forall N \geq \overline{N}$, the optimal price $p^* \geq V_k$. That is, $\lim_{N\to\infty} p_N^* = V_\infty = 1$.

The results also imply that we can construct a sequence of prices $\{\underline{p}(N)\}$ that bound the optimal price from below for each N. This sequence is weakly increasing and converges to one. Indeed, in an earlier version of the paper, we show that the lower bound increases with N following a power law. □

F. Proof of Proposition 4

PROOF: According to Lemma 1, the proposer is never able to cover the cost when no threshold is set (equivalently, when $T = 1$). For the second part, it is sufficient to consider the case $p = V_N$ and $T = N$. With positive probability, all agents receive a positive signal, in which case all projects with a cost not exceeding V_N are financed. Since agents' posterior cannot exceed V_N after any history of actions, projects with $\nu > V_N$ cannot be financed through the threshold implementation. □

G. Proof of Proposition 5

PROOF: From the standard Gambler's Ruin problem we know that as $N \to \infty$, for a given V_k, the conditional probability that an UP cascade occurs at a finite time is one if $V = 1$ and $\frac{(1-q)^{k+1}}{q^{k+1}}$ if $V = 0$ (Feller (1968), p. 347, equation (2.8)). Given the result that $\lim_{N\to\infty} p^* = 1$, the corresponding $\bar{k}(p)$ goes to infinity. Then,

$$\lim_{N\to\infty} \mathcal{P}^{II} = \lim_{N\to\infty} \Pr(A_N \geq T|V = 0) = \lim_{N\to\infty} \left(\frac{1-q}{q}\right)^{\bar{k}(p)+1} = 0. \tag{A.15}$$

We next prove $\lim_{N\to\infty} \mathcal{P}^I = 0$. Suppose that $\lim_{N\to\infty} \mathcal{P}^I \geq \omega$ and consider a price $p = 1 - \omega$. Then, the proposer's expected (average) profit is $p(\Pr(A_N \geq T(p)|V = 1) + \Pr(A_N \geq T(p)|V = 0))\mathbb{E}(A_N|A_N \geq T(p))$. It follows that

$$\begin{aligned}
\lim_{N\to\infty} \pi(p, T(p), N) &= \lim_{N\to\infty} p\big(\Pr(A_N \geq T(p)|V = 1) \\
&\quad + \Pr(A_N \geq T(p)|V = 0)\big)\frac{\mathbb{E}(A_N|A_N \geq T(p))}{N} \\
&> \lim_{N\to\infty} p\Pr(A_N \geq T(p)|V = 1)\frac{\mathbb{E}(A_N|A_N \geq T(p), V = 1)}{N} \\
&\geq \lim_{N\to\infty} p^*(1-\omega)\mathcal{P}^I)\frac{\mathbb{E}(A_N|A_N \geq T(p^*), V = 1)}{N} \\
&= \lim_{N\to\infty} \frac{\pi(p^*, T(p^*), N)}{N},
\end{aligned} \tag{A.16}$$

where the first strict inequality comes from the fact that for finite p and associated k, $\lim_{N\to\infty} \Pr(A_N \geq T(p)|V = 0) = (\frac{1-q}{q})^{\bar{k}(p)+1} > 0$. However, we previously proved that the optimal price $\lim_{N\to\infty} p^* = 1$, a contradiction. We thus conclude that $\lim_{N\to\infty} \mathcal{P}^I = 0$. □

H. Proof of Proposition 6

PROOF: To begin, we prove that $\mathbb{E}[V|A_N]$ is weakly increasing when $A_N < T$. Here, we use T to denote the AoN threshold, which could be either exogenous or endogenous.

(1) $A_N < T - 1$: According to (A.6), all agents act based on their private signals when $A_{i-1} < T - 1$ (and no immediate cascade follows) and $k_{i-1} \leq \bar{k}(p)$. Therefore, $\mathbb{E}[V|A_N]$ is strictly increasing with A_N when there is no UP cascade, and the evaluation is strictly less than $V_{\bar{k}(p)+1}$. Moreover, we know that if an UP cascade starts after some agent i in the queue, then the total support is at least $\frac{i+\bar{k}(p)+1}{2} + N - i > \frac{N+\bar{k}(p)+1}{2}$. Therefore, whenever there is a UP cascade, there are always more supporting agents compared to when there is no UP cascade.

(2) $A_N = T - 1$: In this case, with an argument similar to the previous case, we can show that there are at least $T - 1$ positive signals if there is no UP cascade. However, it is possible that a DOWN cascade starts after the $T - 1$'th supporter, which implies that the expected value stays at a value not exceeding $V_{\bar{k}(p)-2}$ for the agents in the cascade, including the last agent. Since $\bar{k}(p) - 2 \geq 2(T - 1) - N$, the expected value in this case is strictly bounded below by $V_{\bar{k}(p)-2}$, which implies that the increase from $A_N = T - 2$ to $A_N = T - 1$ strictly improves the publicly perceived valuation.

By comparing the posteriors in the cases mentioned above, we can get that $\mathbb{E}[V|\mathcal{H}_N]$ is weakly increasing in A_N. Case 1 also indicates that $\mathbb{E}[V|\mathcal{H}_N, A_N < T^*]$ is strictly increasing in A_N. With exogenous AoN, a potential UP cascade in Case 1 simply means that A_N weakly improves the perceived valuation; for $A_N \geq T$, the project is implemented and more supporting decisions obviously weakly increase the posterior valuation. Therefore, the statement that $\mathbb{E}[V|\mathcal{H}_N]$ is weakly increasing in A_N holds generally for threshold T. □

I. Discussion on Option to Wait and Proof of Proposition 7

Model Characterization: To extend our baseline model, consider agent i, who first arrives in period i, and denote her action in each period $t \geq i$ by $a_i^t \in \{-1, 0, 1\}$, where zero indicates that agent i delays her decision in period t to the next period which is a feasible action only when $i = t$ or $a_i^{t-1} = 0$, that is, she has not supported or rejected the project yet. In any period t, after agent t's decision, all agents already waiting from earlier periods make decisions one at a time (ordered by their first arrival time). For ease of exposition, if agent i chooses not to wait at time t, $a_i^t \neq 0$, we write $a_i^l = a_i^t$, $\forall\, l > t$. Assumptions 1 and 2 still apply.

With the option to wait, for agent i at period $t \geq i$, the history can be summarized as

$$\mathcal{H}_i^t = \begin{cases} \left(a_1^1, a_2^2, a_1^2, a_3^3, a_1^3, a_2^3, \ldots, a_{t-2}^{t-1}, a_t^t\right) & \text{if } i = t \\ \left(a_1^1, a_2^2, a_1^2, a_3^3, a_1^3, a_2^3, \ldots, a_{t-2}^{t-1}, a_t^t, a_1^t, a_2^t, \ldots a_i^t\right) & \text{if } i < t, \end{cases} \tag{A.17}$$

and A_i^t can be defined as

$$A_i^t = \begin{cases} \sum_{1\leq j\leq t-1} a_j^{t-1} \mathbb{1}_{a_j^{t-1}=1} + a_t^t \mathbb{1}_{a_t^t=1} & \text{if } i = t \\ \sum_{i+1\leq j\leq t-1} a_j^{t-1} \mathbb{1}_{a_j^{t-1}=1} + \sum_{1\leq j\leq i} a_j^t \mathbb{1}_{a_j^t=1} + a_t^t \mathbb{1}_{a_t^t=1} & \text{if } i < t. \end{cases} \tag{A.18}$$

Proof of Proposition 7: PROOF: For any given pair of (p, T), there exists an equilibrium with strategy profile a_i^{j*} and posteriors $P(V = 1|\mathcal{H}_{i-1}^i) = V_{k^*(\mathcal{H}_{i-1}^i)}$, where

$$a_i^{i*} = \begin{cases} \mathbb{1}_{\{x_i=1\}} & \text{if } A_{i-1}^{i-1} < T-1,\ k_{i-1} \leq \bar{k}(p),\ i < N \\ 1 & \text{if } k_{i-1} > \bar{k}(p) \\ 0 & \text{if } A_{i-1}^{i-1} \geq T-1,\ k_{i-1} < \bar{k}(p) - 1 \\ \mathbb{1}_{\{x_i=1\}} & \text{if } A_{i-1}^{i-1} \geq T-1,\ k_{i-1} \in \{\bar{k}(p), \bar{k}(p)-1\} \end{cases} \tag{A.19}$$

and for $j > i$,

$$a_i^{j*} = \begin{cases} 2\mathbb{1}_{\{k_N \geq \bar{k}(p)\}} - 1 & \text{if } j = N \ \ \& \ \ a_i^{N-1} = 0 \\ a_i^{i*} & \text{otherwise,} \end{cases} \tag{A.20}$$

$$k_i^*(\mathcal{H}_{i-1}^i) = \begin{cases} k_{i-1} + \left(2a_i^i - 1\right) & \text{if } A_{i-1}^{i-1} < T-1,\ k_{i-1} \leq \bar{k}(p),\ i < N \\ k_{i-1} & \text{if } \ k_{i-1} > \bar{k}(p) \\ k_{i-1} & \text{if } A_{i-1}^{i-1} \geq T-1, \quad k_{i-1} < \bar{k}(p) - 1 \\ k_{i-1} + (2a_i^i - 1) & \text{if } A_{i-1}^{i-1} \geq T-1, \quad k_{i-1} \in \{\bar{k}(p), \bar{k}(p)-1\}. \end{cases} \tag{A.21}$$

To see these results, suppose agent i observes $x_i = 1$, and she has no incentive to deviate. If she chooses rejection or waiting, then all subsequent agents misinterpret her action and update their beliefs as if $x_i = -1$. This would result in failure for some projects that should be financed if i correctly reveals her information.

If agent i observes $x_i = -1$, as we discuss in the baseline model, if there is an UP cascade she chooses to invest. When there is no UP cascade yet, she has no incentive to invest and waiting is a weakly dominating strategy since she can always reject later. Thus, her first action of waiting still reveals her information. □

J. Proof of Proposition 8

PROOF: When agents have options to wait, investors with negative signals invest if the project is expected to be implemented. Thus, the proposer is effectively maximizing $(p-\nu)\Pr(A_N \geq T)$. Also notice that when N goes to infinity, the chance that the project will be implemented without an UP cascade goes to zero. From the proof for Corollary 3, we have for any k

$$\lim_{N\to\infty}(V_k-\nu)\sum_{i=1}^{N-1}\varphi_{k+1,i+1} \geq \lim_{N\to\infty}(V_{k-1}-\nu)\sum_{i=1}^{N}\varphi_{k,i}.$$

The optimal price p therefore goes to one when N goes to infinity. Because $\Pr(x_i=1|V=1)=q>1-q$, (Feller, 1968), p. 347, equation (2.8), shows that the probability that an UP cascade takes place for some finite agent is one when $V=1$ and $\frac{(1-q)^{\bar{k}(p)+1}}{q^{\bar{k}(p)+1}}$ when $V=0$. So all good projects will be implemented almost surely when N goes to infinity, and bad projects will be abandoned almost surely as p goes to one when N goes to infinity. Because agents with negative signals will wait until an UP cascade and good projects will always generate a high-enough public valuation for an UP cascade, even agents with negative signals will invest. The scale of the project is thus efficient as well. □

K. Proof for Lemma 2

PROOF: Any equilibrium involves a subgame-perfect equilibrium following the proposer's decision on p and T. We need only to show that any subgame-perfect equilibrium is either the informative one characterized in Proposition 1 or one that involves a group of free riders whose actions before a cascade are ignored.

For any agent observing a high signal, it is her dominating strategy to contribute when there is a positive probability of reaching the AoN threshold (and her action would be irrelevant if the project would not be implemented for sure). For an agent observing a low signal, if she knows that in equilibrium the subsequent agents update their beliefs based on her action, then she always rejects as discussed in the proof for Proposition 1. However, if she knows that her support for the proposal does not positively update subsequent agents' posteriors, then for any rational off-equilibrium belief (i.e., if subsequent agents observe a rejection instead of a supporting action, they do not positively update their beliefs), supporting becomes a dominant action since it allows her to free ride on the gatekeeper's decision. Hence, she would support the proposal.

Having proven that a PBNE is either an informer equilibrium or a free-rider equilibrium, we next show that if $p \in \{V_k, K=-1,0,\ldots N\}$, then in any free-rider subgame-perfect equilibrium, the project would be implemented only if there is an UP cascade. Suppose agent i is a free rider (given the history $\mathcal{H}_{i-1}$) in a free-rider subgame-perfect equilibrium. Then $a_i=1$. Otherwise, suppose to the contrary that there exists a free-rider subgame-perfect equilibrium in which agent i always rejects the project. Then, the project will be implemented only when the public posterior is at least p after the T^{th} supporting agent. However, if agent i's private signal is $x_i = 1$, then her conditional posterior is strictly higher than p, suggesting that she has incentive to deviate.

If agent i always supports the project, then she must have no incentive to deviate when her private signal is $x_i = -1$. Given $x_i = -1$, if the public posterior after the T^{th} supporting agent is $V_{\bar{k}(p)+1}$, then agent i's conditional posterior is $V_{\bar{k}(p)} = p$. If the public posterior after the T^{th} supporting agent is $V_{\bar{k}(p)}$, then agent i's conditional posterior is $V_{\bar{k}(p)-1} < p$. For agent i, the following inequality must hold for her individual rationality of investing,

$$\varphi(V_{\bar{k}(p)} - p) + Q(V_{\bar{k}(p)-1} - p) = Q(V_{\bar{k}(p)-1} - p) \geq 0, \tag{A.22}$$

where φ is the probability that the public posterior after the T^{th} supporting agent is $V_{\bar{k}(p)+1}$ and Q is the probability that the public posterior after the T^{th} supporting agent is $V_{\bar{k}(p)} = p$, conditional on the history $\mathcal{H}_{i-1}$ and agent i's private observation $x_i = -1$. In other words, conditional on $x_i = -1$, agent i breaks even when the public posterior after the T^{th} supporting agent is $V_{\bar{k}(p)+1}$ and loses money when the public posterior after the T^{th} supporting agent is $V_{\bar{k}(p)}$. For the agent to be rationally free riding by always supporting, it must be the case that $Q = 0$, that is, the project is implemented with an UP cascade.

We next show that for every investor, the informer subgame-perfect equilibrium weakly Pareto-dominates all free-rider subgame equilibria. In a free-rider subgame-perfect equilibrium, for each realization path $x \in X$ that results in project implementation, let h_T be the T^{th} supporting agent. Consider a corresponding $x'(x)$: If there exists a sequence of free riders $\{j_1, j_2, \ldots\}$, then move the sequence of signals $\{x_{j_1}, x_{j_2}, \ldots\}$ to the right of signal x_{h_T}. For each free-rider subgame-perfect equilibrium, $x'(x)$ is an injective function that maps each sequence x to a distinct sequence $x'(x)$ such that $x'(x)$ is a realization path that results in an UP cascade (and thus project implementation) in the informer subgame-perfect equilibrium. Now consider agent i in the free-rider subgame-perfect equilibrium. If she observes $x_i = -1$, she breaks even if she is a free rider, and she also receives zero when she is either an informer or in an UP cascade. Next suppose that agent i observes $x_i = 1$. For each realization path x that results in the project implementation, she always chooses the same action and receives the same payoff on the corresponding realization path $x'(x)$ in the informer subgame-perfect equilibrium. Since x and $x'(x)$ are equally likely to happen, Agent i is weakly better off in the informer subgame-perfect equilibrium.

Finally, suppose a free-rider subgame-perfect equilibrium involves at least two free riders. Then, with positive probability there exists a sequence x with at least two free riders all observing positive private signals and the project is implemented. Similar to the discussion above, on the realization path x, just after the $(T-1)^{\text{th}}$ supporting agent the public posterior is $V_{\bar{k}(p)}$. Notice that the $(T-1)^{\text{th}}$ supporting agent must be the $(N-1)^{\text{th}}$ agent, as otherwise $Q > 0$. In the free-rider subgame-perfect equilibrium, consider the following realization path $\hat{x}(x)$: the first $N-1$ signals are the same as those in sequence x, while $x_N = -1$. Given the sequence $\hat{x}(x)$, the project would not be implemented in the free-rider subgame-perfect equilibrium, but would be implemented in the informer subgame-perfect equilibrium. Moreover, given at least two more positive signals, all investing agents receive positive expected profit on the sequence $\hat{x}(x)$. Therefore, this free-rider equilibrium is strictly dominated by the informer equilibrium. The proposition follows. □

L. Proof for Proposition 9

PROOF: Clearly, for equilibria with only one free rider, one can simply view the case as that of an informer equilibrium with $N-1$ agents. As $N \to \infty$, the resulting equilibria converge. Therefore, we need only to focus on free-rider equilibria with at least two free riders.

As discussed in the proof of Lemma 2, in particular equation (A.22), the unique equilibrium is an informer equilibrium when $p \in \{V_k, k = -1, 0, \ldots N\}$. The proposer's per-investor profit $\frac{1}{N}\pi(p, T)$ should be at least $\frac{1}{N}\pi(p^*(N), T^*(N))$, which is the profit when the proposer optimizes over $p \in \mathbb{Z}^+ \cup \{-1\}$.

Proposition 5 for $\{(p^*(N), T^*(N)\}_{N=1}^{\infty}$ implies that with any optimal path of $\{(p(N), T(N)\}_{N=1}^{\infty}$, project implementation efficiency is achieved and the proposer extracts all of the surplus as N goes to infinity. This condition requires $p_N \to 1$ and that both error probabilities go to zero, as shown in Proposition 5.

Next, let the number of informers in a subgame-perfect equilibrium E be $Z_N^E(p(N), T(N))$ when the proposer's endogenous design is $(p(N), T(N))$. Then, for any positive integer l, $\lim_{N\to\infty} Pr(Z_N^E(p(N), T(N)) < l) \to 0$.

To see this, we again need only to consider free-rider equilibria with at least two free riders. For a given N, the corresponding proposal $(p(N), T(N))$, subgame-perfect equilibrium E, and sequence of signals x define $Z_N^E(x; (p(N), T(N)))$ as the total number of informers for sequence $x \in X$. Hence, we need to show $\Pr(Z_N^E(x; (p(N), T(N)) < l) \to 0$, where the probability is taken over $x \in X$.

Consider the contrary, and suppose for some $\varepsilon > 0$, $\Pr(Z^E(x; (p(N), T(N))) < l) > \varepsilon$ for infinite values of N. For such an N, we have

$$\frac{\pi^E(p(N), T(N))}{N} < \frac{1}{2Np(N)}\pi_1^E(p(N), T(N))$$
$$< \frac{1}{2Np(N)}(1 - \varepsilon(1-q)^l)Np(N)(1-\nu) = \frac{1-\varepsilon(1-q)^l}{2}(1-\nu),$$

where $\pi_1^E(p(N), T(N))$ denotes the proposer's profit when the project is good. The first inequality holds because an agent should assign probability of at least $p(N)$ on the project being good to be willing to pay $p(N)$. Since this holds for all agents and after all the histories, the proposer's gross revenue when $V = 0$ cannot exceed $\frac{1-p(N)}{p(N)}$ times that when $V = 1$. Therefore,

$\frac{1-p(N)}{2Np(N)}(\pi_1^E(p(N), T(N)) + \nu) \geq \frac{1}{2N}(\pi_0^E(p(N), T(N)) + \nu)$, which can be simplified to get the first inequality, since $\nu \geq 0$.

The second inequality follows from the fact that the proposal is not accepted when all of the informers receive a low signal. But in the proof of Proposition 9, we show that $\frac{\pi^{E_N}(p(N),T(N))}{N}$ goes to $\frac{1}{2}(1-\nu)$ in this sequence of numbers, which is a contradiction.

With this, there exists infinitely many informers almost surely. All results can be then shown similarly as in the proof of Proposition 5. □

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.
Replication Code.