



RIM-based value premium and factor pricing using value-price divergence

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ABSTRACT

We document that value-to-price, the ratio of Residual-Income-Model-based valuation to market price, subsumes the power of book-to-market ratio and many other value or quality measures in predicting stock returns. Long-short value-to-price portfolios hedge against momentum, revitalize the seemingly missing value premium over past decades, and generate significant returns after adjusting for common factors. The value-price-divergence (VPD) factor constructed from the average returns of these portfolios within small and big stocks is not spanned by these known factors. Max Sharpe ratio and constrained R-squared tests reveal that VPD is a better substitute for the traditional value factor and that a four-factor model using the VPD, market, momentum, and size factors outperforms most extant benchmarks in explaining the cross-section of expected equity returns, including value-to-price portfolios as test assets. The findings remain robust under alternative specifications of equity cost of capital.

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1. Introduction

Value investing has been pervasive in finance and the evidence for positive value premiums around the globe spans over half a century (e.g., Fama and French, 1998, 2012; Davis et al., 2000; Asness et al., 2013). The theoretical premise behind value investing is that paying lower market prices for assets with higher intrinsic

value (calculated through discounting expected future cash flows) should provide higher expected returns. However, value premiums in recent data are insignificant or negative, eroding the belief and confidence in value investing. For example, although the authors view it as random, Fama and French (2021) find a much lower value premium, if any, in the United States in 1992–2019 than in 1963–1991 (Fama and French, 1992). In fact, the true intrinsic values of assets are unobserved, and book-to-market ratios may not serve as an effective metric for value investing at all, a point first made by Graham et al. (1934).

To construct a direct proxy for the fundamental values of assets, we utilize forward-looking analyst forecasts and the residual income model (RIM, e.g., Frankel and Lee, 1998; Lee et al., 1999) to provide a valuation, V , of a firm's equity share. We then compute the monthly "value-to-price" ratio of V over the stock price P , which contains different information than the book-to-market ratio. In the traditional approach (e.g., Fama and French, 1992), the equity value is derived from the current financial statements of the firms. In our approach, the equity value is a forward-looking measure that accounts for the firms' future ability to pay shareholders

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in excess of their opportunity costs.²

We sort stocks based on the ratio to construct long-short portfolios, and document a new and robust RIM-based value premium that potentially reflects firms' intangibles, real options, etc. A value-weighted portfolio can generate over 7% annualized return over the past 40 years. We then construct a value-price divergence (VPD) factor using V/P-sorted portfolios to price the cross-section of expected stock returns. We find that a four-factor model using the VPD, market, momentum, and size factors outperforms major extant benchmarks. Regardless of whether the RIM-based value premium represents a rational pricing of risk or simply mispricing, investors can use it to devise strategies and asset pricers may substitute the conventional value factor with VPD, especially when working on recent data.

Specifically, we find that the value-to-price ratio (V/P) strongly predicts future stock returns: during the 40-year period from July 1978 to June 2018, the long-short portfolio that buys the underpriced (high V/P) stocks and shorts the overpriced (low V/P) stocks generates a significant monthly raw return of 0.61% and significant monthly alphas ranging from 0.59% to 0.89% with respect to popular factor models, such as the Hou et al. (2021) q5-factor model, Fama and French (2015) 5-factor model, Asness et al. (2019) 5-factor model, Stambaugh and Yuan (2016) 4-factor model, and Barillas and Shanken (2018) 6-factor model. These results are robust after we control for common firm characteristics, such as size, book-to-market, operating profit, investment, and momentum. The findings are corroborated with Fama-Macbeth regressions that simultaneously control for various firm characteristics. While V/P is correlated with book-to-market in the early sample period, they diverged significantly during the past few decades: V/P earned a premium several times higher than the traditional value premium.³

We then construct a value-price-divergence (VPD) factor as the equal-weighted average returns of the long-short V/P portfolios within small and big stocks and relate it to popular pricing factors. Since V/P is fundamental-based and highly correlated with past performance, we focus on the momentum factor and fundamental-based factors in the Asness and Frazzini (2013) 3-factor model, Fama and French (2015) 5-factor model, the Hou et al. (2021) q5-factor model, the Asness et al. (2019) model, and the two mispricing factors in the Stambaugh and Yuan (2016) 4-factor model. We find that the VPD factor has significantly negative loadings on the market, size, and momentum factors, and positive loadings on value, profitability, and quality-minus-junk (QMJ) factors.

We then employ the max squared Sharpe ratio tests proposed by Barillas and Shanken (2017) and the constrained R-squared method proposed by Maio (2019) – two recent improvements over the plain vanilla GRS test (Gibbons et al., 1989) in assessing empirical asset pricing models – to examine whether factor models built on VPD factor better explain cross-sectional stock returns. The max squared Sharpe ratio tests confirm that replacing HML with VPD in well-established models, such as the Asness and Frazzini (2013) 3-factor and Carhart (1997) 4-factor models, substantially improves the explanatory power of the model. A 4-factor model that includes the MKT, SMB, VPD, and UMD factors has a max squared Sharpe ratio comparable to the Fama and French (2015) 5-factor model, and a 6-factor model that includes the MKT, SMB, VPD, CMA, RMW, and UMD factors produces the highest max squared

Sharpe ratio among all models considered. The constrained R-squared test confirms that the four-factor model using MKT, SMB, VPD, and UMD performs even better than well-established multi-factor models, such as the Fama and French (2015) 5-factor, Stambaugh and Yuan (2016) 4-factor, Asness et al. (2019) 5-factor, and Hou et al. (2021) q5-factor models. Overall, the findings suggest that our intrinsic valuation captures firm fundamentals more comprehensively than book value and is useful in cross-sectional asset pricing.

Our paper adds to the literature documenting financial ratios predicting stock returns, especially studies concerning the value premium. We use an accounting-based valuation that takes into consideration both quality and relative cheapness (Novy-Marx, 2013a, 2013b; Asness et al., 2019). Our approach builds on the seminal contributions of Frankel and Lee (1998) and Lee et al. (1999), which explore RIM properties for understanding cross-sectional or time series stock returns. We add by relating the V/P ratio predictability to the value premium and a new factor model for cross-sectional asset pricing.⁴ We also (i) focus on a different horizon of predictability (months as opposed to years), (ii) allow for cross-sectional variations in the discount rates, and (iii) cover observations over the past two decades when the value premium appears to decline.⁵

We are among the earliest to use an alternative metric for an asset's intrinsic worth to better understand value investing and the decline in value premium over the past few decades. Moreover, we are the first to utilize the VPD factor to explain the cross-section of stock returns and among the first to point out the limitation of using the book-to-market ratio in value investing (e.g., Park, 2019). A related contemporaneous study by Bartram and Grinblatt (2018) documents that the divergence of a firm's peer-implied value estimate from its market value represents mispricing, motivating a profitable convergence trade. Golubov and Konstantinidi (2019) apply the multiples-based market-to-book decomposition of Rhodes-Kropf, Robinson, and Viswanathan (2005) to show that the market-to-value component drives all the value strategy returns. Hou et al. (2020) construct a value signal following the specifications in Frankel and Lee (1998) and our paper, but use a constant 12% discount rate, which results in a declining value premium. Recently, Goncalves and Leonard (2023) analyze the decline in correlation between the book-to-market ratio and a fundamental equity-to-market ratio that they introduce to resurrect the value premium.

Our findings on persistent excess returns of value investing are consistent with and complement these papers by demonstrating that a widely adopted absolute valuation model in accounting, RIM, generates an effective value signal, which resurrects the value premium and constitutes a superior substitute for the value factor for asset pricing. While Goncalves and Leonard (2023) focus on long-term cash flow forecasts through a value-at-risk (VaR) that predicts equity payouts going forward to infinity (and a constant discount rate), we focus on analysts' forward-looking short-term EPS forecasts and on discount rates that vary cross-sectionally.

² The V/P ratio is not very correlated with monthly book-to-market ratio, with a correlation (60-month moving average) going from 0.2 in early 1980s to 0.02 in 2008, back to 0.1 in 2013–2015, and then 0.05 in 2018.

³ Several studies investigate this divergence: book-to-market predicts returns only because it contains information about past earnings (Ball et al., 2020); it may not reflect the changing corporate environment (Kahle and Stulz, 2017), nor does it capture the intangibles which have been growing over the past decade (Peters and Taylor, 2017; Park, 2019; Eisfeldt et al., 2022).

⁴ Frankel and Lee (1998) use I/B/E/S consensus forecasts and a residual income model to estimate a firm's fundamental value and finds that V is highly correlated with contemporaneous stock price and that the V/P ratio is a good predictor of long-term cross-sectional returns. Lee, Myers, and Swaminathan (1999) apply the same technique to study the intrinsic value of the Dow Jones Industrial Average and study the time series relation between value and price. Other related studies include Penman and Sougiannis (1998), Dechow, Hutton, and Sloan (1999), Abarbanell and Bernard (2000), and Ali et al. (2003). Conceptually, accounting-based valuation goes beyond the RIM model, but RIM remains popular both in academic research and commercial products.

⁵ As analysts usually adjust their forecasts whenever new information is received, a more frequent estimation using the latest analyst forecasts each month allows us to more accurately estimate firms' fundamental value.

Our research also adds to asset pricing models with observable factors (e.g., Fama and French, 1993, 2016; Carhart, 1997; Pastor and Stambaugh, 2003; Hou et al., 2014, 2015). Fama and French (2015) add additional investment (CMA) and profitability (RMW) factors into the original 3-factor model and find that the original value factor (HML) becomes redundant. Similarly, Hou et al. (2021) develop a 5-factor q5-model that includes the market, size, investment (investment-to-assets), profitability (ROE), and expected growth factors from the q-theory, and show that the q5-model outperforms the Fama and French (2015) 5-factor model. While the value, investment, and profitability factors only involve the information on firms' financial statements that is stale and backward-looking, our VPD factor, which is based on analysts' consensus forecasts, naturally nest the market professionals' expectations on a firm's future investment and profitability. It is thus not surprising that a factor pricing model using the VPD factor empirically performs better than other models in the max squared Sharpe ratio and constrained R-squared tests.

The remainder of the paper is organized as follows. In Section 2, we present the residual income model and describe the data. In Section 3, we study the relation between V/P and future stock returns. In Section 4, we construct the VPD factor and compare it to the other factors. In Section 5, we compare various factor models based on max squared Sharpe ratio tests and constrained R-squared tests. We check model robustness in Section 6 and conclude in Section 7.

2. Residual income model and data

The main databases that we use are CRSP, Compustat, and I/B/E/S. We use the 3-period residual income model (RIM), also known as the edward-bell-ohlson (EBO) model (see Edwards and Bell, 1961; Ohlson, 1995), where a perpetuity is assumed beyond the third period (year), to estimate the fundamental value V of a firm at the end of each month.

Recall that a stock's intrinsic value, defined as the present value of its expected future dividends (denoted by D) based on currently available information, is of the reduced form:

$$V_t = \sum_{i=1}^{\infty} \frac{E_t[D_{t+i}]}{(1+r_{e,t})^i} = B_t + \sum_{i=1}^{\infty} \frac{E_t[(ROE_{t+i} - r_{e,t}) * B_{t+i-1}]}{(1+r_{e,t})^i}, \quad (1)$$

where we follow the literature to assume a flat term structure of discount rates/cost of equity capital $r_{e,t}$ (e.g., Lee et al., 1999), and E_t denotes time t expectation. The equation holds as long as a firm's earnings and book value are forecasted in a manner consistent with "clean surplus" accounting, which requires that all gains and losses affecting book value are included in earnings (Edwards and Bell, 1961; Ohlson, 1995).

In a 3-period RIM model that is estimated monthly, this would be:

$$V_t = B_{y(t)} + \frac{(FROE_{y(t)+1} - r_{e,t})}{(1+r_{e,t})} B_{y(t)} + \frac{(FROE_{y(t)+2} - r_{e,t})}{(1+r_{e,t})^2} E_t[B_{y(t)+1}] + \frac{(FROE_{y(t)+3} - r_{e,t})}{(1+r_{e,t})^2 r_{e,t}} E_t[B_{y(t)+2}], \quad (2)$$

where:

$B_{y(t)}$ = Book value from the most recent annual statement for fiscal year $y(t)$ at the end of month t , and $y(t)$ is the fiscal year whose annual statement is the most recent available one at the end of month t . The book value is calculated as total assets (Compustat Item 6 AT) minus total liabilities (Compustat Item 181 LT), plus balance sheet deferred taxes (Compustat Item 74 TXDB), plus balance sheet investment tax credit (Compustat Item 208 ITCB), minus the liquidating value of preferred stock (Compustat Item

10 PSTKL) if available, or the redemption value of preferred stock (Compustat Item 56 PLTKRV), or carrying value of preferred stock (Compustat Item 130 PSTK) adjusted for net stock issuance from last fiscal year end to the end of month t . Annual reports are assumed to become available six months after the fiscal year ends, as is standard in the literature.

$r_{e,t}$ = Industry-specific annual cost of equity at the end of month t . Similar to Fama and French (1997), we use four-digit SIC codes to assign firms to 48 industries, and the cost of equity for a firm is the same as that for its industry. An industry's annual cost of equity at the end of month t is 12 times the product of the coefficients of the Fama-French (henceforth "FF") 3-factor model from 60-month rolling regressions of monthly industry excess returns on FF 3-factor returns and the long-term factor premiums, plus the average annual risk-free rate in 1978–2018.

$FROE_{y(t)+i}$ = Forecasted ROE for period $y(t) + i$, $i = 1, 2, 3$. It is calculated as $\frac{FEPS_{y(t)+i}}{B_{y(t)+i-1}}$, where $FEPS_{y(t)+i}$ is the I/B/E/S mean forecasted EPS for year $y(t) + i$, denoted as FYi, and it is announced on the third Thursday of each month t . For $i = 3$, $FEPS_{y(t)+3} = FEPS_{y(t)+2}(1 + Ltg)$, where Ltg is the mean long-term earnings growth forecasted by analysts. When this is missing, we use the composite growth rate implicit between FY1 and FY2 to forecast FY3.

$E_t[B_{y(t)+i}] = E_t[B_{y(t)+i-1}] * [1 + FROE_{y(t)+i} * (1 - k)]$, where k is the current dividend payout ratio and it is equal to dividends-common/income before extraordinary items-adjusted for common stock equivalents if $EBIT > 0$, or equal to total dividends/(0.06* total assets) if $EBIT \leq 0$. Payout ratios that are greater than 1 or less than 0 are treated as missing values. Note that the dynamics of book value essentially involve adding retained earnings to current earnings.

In estimating V , we constrain our sample to common stocks (CRSP share code 10 or 11) of non-financial firms whose closing price at the end of each month are greater than \$5 and remove firms with negative book values. Furthermore, we also eliminate firms with ROE or FROE greater than 100% in order to exclude firms with extremely low book values.

The main inputs of the model are the I/B/E/S mean forecasted earnings. These forecasts capture firms' future profitability and growth opportunities. When these forecasts are not available, we backfill them with the most recent ones in the past 12-month period.

Our monthly V/P ratio is defined as the fundamental value V divided by the market capitalization, both calculated at the end of each month. To avoid bias caused by potential data errors in the I/B/E/S dataset, we winsorize V/P ratios to 98% at each month-end. Furthermore, to compare with 12 popular firm characteristics, we also compute the following variables at the end of month t : annual market cap (ME), monthly market cap (ME_M), annual book-to-market ratio (B/M), monthly book-to-market ratio (B/M_M), operating profitability (OP), investment (Inv), net stock issuance (NS), accruals to book value (Ac/B), β (Beta), the variance of daily total returns (Var), the variance of daily residuals from the Fama and French (1993) 3-factor model (RVar), momentum (Mom), and the turnover ratio (Turnover). The definitions of these variables are listed in the appendix.

Our final sample spans from July 1978 to June 2018, and covers 7584 firms and 728,939 firm-month observations of V/P. The mean and median of V/P ratios across all stocks are 0.66 and 0.58, respectively. However, this does not necessarily imply that stocks are overpriced on average as we could easily scale the numbers to 1 by adjusting the cost of equity. V/P itself has a right skewness of 1.35, while the distribution of its natural log is symmetric and is similar to a normal distribution (see Fig. 1).

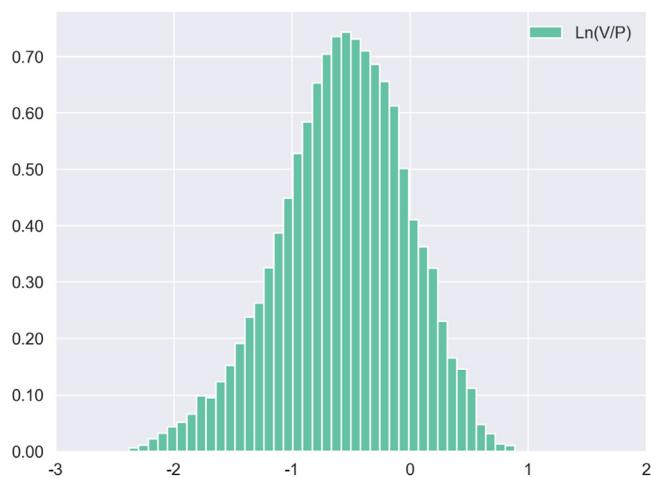


Fig. 1. Distribution of V/P. In this figure, we plot the distribution of the natural logarithm of the V/P ratio for all non-financial common stocks in the intersection of CRSP/Compustat/IBES during the sample period of June 1978 to June 2018. The V/P ratio is the fundamental value V calculated on the month-end using a 3-period RIM model divided by the market cap on the month-end. The data include analyst forecast consensus earnings per share, dividend payout ratio, industry specific cost of equity, and book value, and are organized by Eq. (2).

3. V/P ratio and stock future returns

To study the relation between the V/P ratio and future stock returns, we use both portfolio sorts and Fama-MacBeth (1973) regressions for robustness. The advantage of the portfolio approach is that the relation between the V/P ratio and future stock returns is not assumed to be linear, but one cannot simultaneously control for other variables. The Fama-MacBeth approach allows us to control for other variables that may potentially affect stock returns, albeit implicitly assuming linear relations between firm characteristics and stock returns.

3.1. V/P portfolio returns

At the end of each month, we construct V/P single-sorted portfolios, as well as V/P and firm characteristic double-sorted portfolios. We then examine the portfolios' returns in the next month. To be included into a month-end portfolio, a stock must have a non-missing return at the end of the formation month.

All portfolio returns are value-weighted returns where the weight is each stock's month-end market cap.⁶ Our first portfolio is constructed at the end of June 1978, and we report portfolio returns for the 480-month period starting from July 1978 and ending in June 2018.

3.1.1. V/P single-sorted portfolios

With single sorting, we split stocks into 10 deciles according to their V/P ratios; the cut-off points for deciles are based on the V/P ratios of stocks on the NYSE. Decile 1 represents the 10% of stocks with the lowest V/P values (most overpriced stocks) and decile 10 represents the 10% of stocks with the highest V/P values (most underpriced stocks).

Fig. 2 shows the convergence of each decile's average V/P to its equilibrium level during the three years after the portfolio is constructed. Over time, the V/P ratio of underpriced stocks moves upwards to its equilibrium level, while that of overpriced stocks

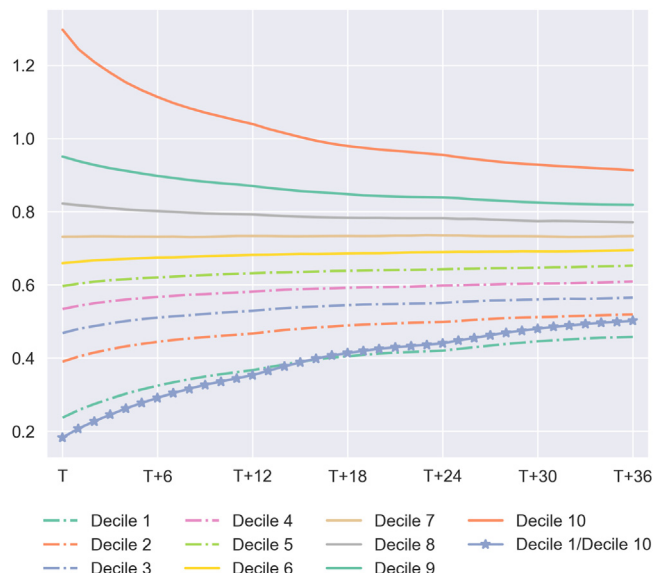


Fig. 2. Post-portfolio-formation V/P Evolution for V/P-sorted deciles. The horizontal axis is the time horizon. T represents the portfolio construction month, and T + k means k month after the portfolio construction. The vertical axis is the equal-weighted average of V/P ratio across stocks within each V/P decile. V/P ratio is defined as the fundamental value V calculated on the month-end using a 3-period Residual Income Model (Eq. (2)) divided by the market cap on the month-end. Decile 1 (10) includes the 10% stocks with the lowest (highest) V/P ratios. Decile 1/Decile 10 is the ratio of decile 1's V/P over decile 10's V/P. The sample includes all non-financial common stocks in the intersection of CRSP/Compustat/IBES during the period June 1978-June 2018.

moves downwards. The convergence is faster in the first year than in the subsequent two years. We also calculate the ratio of decile 1's V/P over decile 10's V/P to represent the market's aggregate "divergence from fundamental." This ratio also shows a convergence over time, reflecting arbitrage forces.

Table 1 shows the time series averages of the cross-sectional averages of various firm characteristics in each decile and the time series average of cross-sectional correlations between a V/P portfolio decile assignment and other firm characteristics' decile assignments. The results indicate that the V/P ratio is mostly correlated with the monthly book-to-market ratio, annual book-to-market ratio, and the past performance, with portfolio correlations of 0.26, 0.17 and -0.17, respectively. Furthermore, firms' characteristic correlations have the same pattern as the portfolio correlations, and both have decreasing trends over time. In particular, the 40-year average of cross-sectional correlations between the V/P ratio and the monthly book-to-market ratio is 0.1, but its rolling 60-month average dropped from 0.2 in early 1980s to 0.02 in June 2008, then rose to 0.1 in 2013–2015, and then dropped to 0.05 in June 2018. High (low) V/P stocks are usually value (growth) stocks and past losers (winners).

While this is not surprising since book-to-market value is included in the calculation of V and since P is related to past performance, its implications are potentially profound. To the extent that V captures the fundamental value of assets, book-to-market is only a noisy approximation for V/P when it comes to value investing. Because the noise in the approximation increased over time, it is not surprising to observe an apparent decline in the value premium if one still focuses on book-to-market. This is a point also belabored in Gonçalves and Leonard (2023), who introduce another cash flow-based fundamental equity metric, which they find drives the value premium.

Also, V/P is positively correlated with profitability (OP) and accruals (Ac/B), and negatively correlated with beta (Beta), suggest-

⁶ It is well-known that equal-weighting tends to bias towards better performance is not as implementable. Our results become even more economically significant with equal weights.

Table 1
Firm Characteristics of V/P Portfolios: 1978–2018.

V/P Decile	V/P	ME	ME_M	B/M	B/M_M	OP	Inv	NS	Ac/B	Beta	Var	RVar	Mom	Turnover
1(Low)	0.25	2942.94	3237.18	0.68	0.59	15.42	18.77	5.83	1.19	1.56	12.18	12.04	34.11	0.75
2	0.41	3500.66	3687.10	0.60	0.52	24.83	16.85	3.60	1.28	1.40	9.06	8.91	29.60	0.65
3	0.49	3706.26	3847.82	0.62	0.54	26.89	15.25	2.94	1.63	1.34	8.18	7.95	24.66	0.61
4	0.56	4209.12	4385.12	0.64	0.57	27.51	14.85	2.81	1.32	1.30	7.95	7.74	22.02	0.60
5	0.62	4212.51	4371.49	0.67	0.61	27.78	14.66	2.73	1.66	1.26	7.77	7.62	19.55	0.60
6	0.68	4320.54	4462.24	0.70	0.65	27.82	14.64	2.89	1.44	1.23	7.74	7.61	17.58	0.59
7	0.76	4614.91	4725.65	0.73	0.70	27.69	14.89	3.05	1.62	1.20	7.82	7.76	15.28	0.59
8	0.85	4632.65	4689.71	0.76	0.74	27.70	15.88	3.24	1.77	1.17	8.04	8.01	13.29	0.60
9	0.97	4642.26	4629.79	0.79	0.79	27.61	16.39	3.85	1.85	1.15	8.57	8.58	11.53	0.61
10(High)	1.32	4036.03	3913.38	0.82	0.87	26.01	20.88	5.67	2.65	1.32	12.01	12.36	7.06	0.73
Correlation with V/P	1.00	-0.02	-0.05	0.17	0.26	0.09	0.01	-0.01	0.03	-0.12	-0.07	-0.05	-0.17	-0.02

This table presents time series averages of cross-sectional averages of various firm characteristics for stocks in deciles sorted by V/P ratios at the end of each month during July 1978–June 2018. V/P ratio is defined as the fundamental value V calculated on the month-end using a 3-period Residual Income Model (Eq. (2)) divided by the market cap on the month-end. The last row lists the cross-sectional correlation between stocks' V/P portfolio assignments and other firm characteristic portfolio assignments. The definitions for ME, ME_M, B/M, B/M_M, OP, Inv, NS, Ac/B, Beta, Var, RVar, Mom, and Turnover are listed in the appendix. ME and ME_M are expressed in million dollars, V/P, B/M, B/M, Beta are expressed in decimals, OP, Inv, Ac/B, Mom, and Turnover are expressed in percentages, and Var and RVar are expressed in base points.

Table 2
Raw Returns for Single-Sorted Portfolios: 1978–2018.

Decile	V/P	ME	ME_M	B/M	B/M_M	OP	Inv	NS	Ac/B	Beta	Var	RVar	Mom	Turnover
1 (low)	0.92	2.01	2.09	1.03	1.08	1.13	1.27	1.30	1.29	1.07	0.96	1.06	0.84	1.03
2	1.06	1.48	1.60	1.15	1.07	0.84	1.29	1.11	1.27	1.09	1.12	1.13	0.96	1.03
3	0.86	1.38	1.39	1.17	1.09	1.01	1.12	1.13	1.12	1.14	1.10	1.15	1.03	1.06
4	0.98	1.25	1.33	1.08	1.13	1.20	1.12	1.07	1.13	1.17	1.10	1.12	0.99	1.17
5	1.11	1.23	1.23	1.17	1.10	0.97	1.15	1.12	1.08	1.01	1.17	1.15	0.99	1.11
6	1.27	1.15	1.18	1.08	1.14	1.08	1.07	1.05	1.02	1.22	1.15	1.16	1.01	1.12
7	1.32	1.23	1.19	1.23	1.20	1.06	1.11	1.14	1.01	1.14	1.30	1.24	1.03	0.99
8	1.17	1.18	1.13	1.15	1.13	1.14	1.13	1.33	1.13	1.05	1.22	1.18	1.18	1.02
9	1.35	1.14	1.13	1.18	1.30	1.21	1.20	1.06	0.96	1.04	1.25	1.17	1.20	1.19
10 (high)	1.53	1.00	1.00	1.39	1.38	1.15	1.05	0.82	0.90	1.20	1.08	0.96	1.57	1.25
High-Low	0.61***	-1.01***	-1.09***	0.35*	0.30	0.02	-0.23	-0.48***	-0.39***	0.13	0.13	-0.10	0.73**	0.22

This table shows the monthly raw returns of ten deciles sorted by various firm characteristics. At the end of each month from June 1978 to June 2018, stocks are split into ten deciles according to the ranking of each firm characteristic. Decile 1 (10) includes the 10% stocks with the lowest (highest) firm characteristics, and a long-short portfolio High-Low that buys stocks in decile 10 and shorts stocks in decile 1 is also constructed at the same time. Each portfolio is then held for 1 month, and its average monthly value-weighted return (percent) is presented in this table. The V/P ratio is defined as the fundamental value V calculated on the month-end using a 3-period Residual Income Model (Eq. (2)) divided by the market cap on the month-end. The definitions for ME, ME_M, B/M, B/M_M, OP, Inv, NS, Ac/B, Beta, Var, RVar, Mom, and Turnover are listed in the appendix. For the High-Low long-short portfolio *, **, *** in "High-Low" row indicate significance at 10%, 5%, 1% level, respectively. All standard errors are calculated using White (1980) t-stats.

ing that firms with high profitability, high accrual, or low beta are more likely to be undervalued (high V/P). On the other hand, V/P is not linear in size, net stock issuance (NS), Var, RVar, and liquidity (Turnover) in the sense that stocks with small market cap, high net stock issuance, high idiosyncratic volatility (Var or RVar), and high turnover tend to have the most extreme V/P, suggesting that the prices of these stocks are more likely to deviate from their intrinsic values.

Table 2 presents the monthly raw returns for portfolios sorted on each firm's characteristics in Table 1. The results indicate that high V/P stocks overperform low V/P stocks, and that the high_minus_low (10–1) long-short portfolios consistently generate a significantly positive raw return over the following month. The raw monthly return for the highest V/P decile (decile 10) is 1.53% while the return decreases to 0.92% for the lowest V/P decile. The 10–1 long-short portfolio generates a 0.61% return, which is significant at the 1% level and corresponds to over 7% annualized returns. As a reference, we confirm the presence of anomalies in size, value, net stock issuance, accruals, and momentum, but we do not observe the profitability, investment, beta, idiosyncratic risk, and turnover anomalies.

This pattern persists when we control for other leading factor models; we run time series regressions of the V/P decile's monthly excess return on various factors from leading factor models, as presented in Table 3, for a one-month holding period. The factor models include the basic CAPM, AF3 (3-factor model with the HML fac-

tor in Fama and French's (1993) replaced by the monthly HML devol factor HMLM, see, Asness and Frazzini, 2013), HMXZ5 (q5-factor model in Hou et al., 2021), FF5 (Fama and French, 2015), FF5+UMD, AFP5 (4-factor model in Carhart, 1997, plus quality-minus-junk factor in Asness et al., 2019), the SY4 (Stambaugh and Yuan, 2016), and BS6 (6-factor model including the three factors in AF3, the momentum factor, and the IA and ROE in HMXZ5, see, Barillas and Shanken, 2018). For most models, deciles 6–10 generate positive alphas, whereas nearly all other deciles generate negative alphas. The 10–1 long-short portfolio generates significantly positive alphas for all factor models; monthly alphas range from 0.61% for the FF5 and BS6 models to 0.89% for the SY4 model.

3.1.2. V/P and firm characteristic double-sorted portfolios

As shown in Table 1, the V/P ratio is correlated with other firm characteristics that have significant impacts on stock returns. To control for firm characteristics that may also affect stock returns, we perform a 5 × 5 double sort on the V/P ratio and firm characteristics. At the end of each month, stocks are dependently split into five V/P ratios according to the ranking of B/M (book-to-market), B/M_M (monthly book-to-market) and Mom (momentum), or independently split by the ranking of ME, ME_M, OP, Inv, NS, Ac/b, Beta, Var, RVar, and Turnover. The cut-off points in each sort are based on NYSE stocks. Since the V/P ratio is highly correlated with book-to-market and past performance, conditional sorts can better control for these two characteristics, and will provide bal-

Table 3
Alphas for V/P Single-Sorted Portfolios: 1978–2018.

Decile	CAPM	AF3	HMXZ5	FF5	FF5 +UMD	AFP5	SY4	BS6
1 (low)	−0.24**	−0.18*	−0.05	−0.10	−0.14	−0.10	−0.25**	−0.01
2	−0.02	0.01	−0.05	−0.08	−0.07	−0.09	−0.04	0.05
3	−0.16*	−0.14	−0.19*	−0.26***	−0.23**	−0.25***	−0.13	−0.16
4	−0.05	−0.03	−0.08	−0.14	−0.12	−0.19**	−0.10	0
5	0.11	0.12	−0.09	−0.04	−0.03	−0.09	−0.06	−0.03
6	0.30***	0.29***	0.21**	0.20**	0.21*	0.17	0.24**	0.20*
7	0.38***	0.36***	0.09	0.17*	0.21**	0.13	0.21**	0.16*
8	0.27**	0.26**	0.11	0.03	0.10	0.06	0.18	0.11
9	0.43***	0.39***	0.2	0.22*	0.28**	0.21	0.32*	0.22
10 (high)	0.53***	0.49***	0.53**	0.52***	0.63***	0.62***	0.65***	0.60***
High-Low	0.77***	0.67***	0.59*	0.61***	0.78***	0.73***	0.89***	0.61***

This table presents the intercepts (alphas in percentage terms) of the time series regressions of monthly excess returns of each V/P sorted portfolio and the long-short portfolio on different factors during July 1978–June 2018: $R_i^t - R_f^t = \alpha_i + \sum_{k=1}^K \beta_{i,k} f_k^t + \epsilon_i^t$, where R_i^t and R_f^t is the month t return of V/P decile i and the risk-free asset, respectively, and f_k^t is the value of kth factor in month t (monthly return for traded factors) in a factor model. In the regression for a long-short portfolio, the dependent variable is the difference between two portfolio returns. Each portfolio is constructed as follows: at the end of each month from June of 1978 to June of 2018, stocks are split into ten deciles according to the ranking of V/P ratio which is defined as the fundamental value V calculated on the month-end using a 3-period Residual Income Model (Eq. (2)) divided by the market cap on the month-end. Decile 1 (10) includes the 10% stocks with the lowest (highest) V/P, and a long-short portfolio High-Low that buys stocks in decile 10 and shorts stocks in decile 1 is also constructed at the same time. Each portfolio is then held for 1 month, and its monthly return is calculated as the value-weighted average of stock returns. The factor models include: basic CAPM, AF3 (Asness and Frazzini (2013) 3-factor model), HMXZ5 (Hou et al. (2021) q5-factor model), FF5 (Fama and French (2015) 5-factor model), FF5+UMD (Fama and French (2015) 5-factor plus Momentum factor model), AFP5 (Carhart (1997) 4-factor plus Asness et al. (2019) Quality-Minus-Junk factor model), SY4 (Stambaugh and Yuan (2016) 4-factor mispricing model), and BS6 (Barillas and Shanken (2018) 6-factor model). *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively. All t-statistics are White (1980) t-statistics.

anced portfolios as well. Quintile 1 (5) includes the 20% of stocks with the lowest (highest) V/P ratio or firm characteristic ratios. We also construct a long-short portfolio V/P 5–1 within each of the five characteristic quintiles that buys stocks in V/P quintile 5 and shorts stocks in V/P quintile 1. We hold each long-short portfolio for one month, and its monthly return is calculated as the month-end market-cap weighted returns of stocks within it.

In Panel A of Table 4, we present each V/P long-short portfolio's monthly raw return. The regression alphas of eight factor models are in Panels B to I. All returns and alphas are in percentages. Panel A shows that the raw monthly returns of the V/P long-short portfolios are significantly positive within most firm characteristic quintiles and the magnitudes range from 0.3% to 0.7%. Specifically, high V/P stocks outperform low V/P stocks in all size (ME and ME_M) quintiles but the largest size quintile, which may be because big stocks are usually very liquid and thus the dispersion in stock returns disappears quickly within a month. A similar pattern is also observed for the turnover ratio (Turnover) and momentum (Mom) quintiles, where the high turnover stocks are also very liquid and “past winner” stocks usually get more attention and thus are more actively traded and liquid.

Controlling for book-to-market ratio (B/M and BM_M), operating profitability (OP), investment (Inv), net stock issuance (NS), accruals (Ac/B), and beta (Beta), the V/P long-short portfolio still generates significantly positive raw returns in most quintiles except quintiles for high and low monthly book-to-market ratios, higher investment, high and medium operating profitability, low accruals, and medium beta. Furthermore, the raw returns are only significantly positive in two of five variance of daily total return (Var) quintiles, suggesting that the V/P ratio may be partially explained by Var or idiosyncratic risk. However, raw return is a noisy measure that does not control for risk factors; thus, we run further regressions of the long-short portfolio's return on factors of commonly used factor models and focus on the alphas of these models.

In addition, Table 4 shows that the V/P ratio exhibits interesting interactions with other asset characteristics but does not provide explanatory power for them. For example, growth stocks are known to have less dispersion in returns that one can easily identify using additional firm characteristics (Piotroski, 2000). Like the

signals in the F-score, the V/P ratio contains information that helps further separate winners from losers for firms with medium and high book-to-market ratios, but not for those with low book-to-market ratios. Indeed, non-linearity and interactions are found to be common and important in recent asset pricing studies employing machine learning (e.g., Freyberger et al., 2020; Cong et al., 2019, 2021) that our findings are consistent with.

Panels B through I in Table 4 list the alphas of the CAPM, AF3, HMXZ5, FF5, FF5+UMD, AFP5, SY4, and BS6 models, respectively. The results suggest that the significantly positive raw returns for the long-short V/P portfolio in Panel A cannot be fully explained by these factors because approximately 65% of the 520 long-short portfolios have significantly positive monthly alphas ranging from 0.3% to 1%. Among all models, HMXZ5 has the most explanatory power for the V/P ratio as 37 of the 65 alphas are statistically insignificant. Overall, our RIM-based value premium interacts strongly with a number of stock characteristics but is not explained by them.

3.2. Fama-MacBeth regression

As an alternative to the portfolio approach, we conduct Fama-MacBeth regressions (Fama and MacBeth, 1973) of individual stock returns on several firm characteristics to assess the statistical significance of the V/P ratio while simultaneously controlling other firm characteristics that may affect stock returns. Specifically, in each month t of our sample period from July 1978 to June 2018, we run the following cross-sectional regression of stock returns on firm characteristics:

$$R_i^t = \alpha^t + \sum_{k=1}^K \beta_k^t X_{i,k}^{t-1} + \epsilon_i^t, \tag{3}$$

where R_i^t is the monthly return of stock i in month t, $X_{i,k}^{t-1}$ is the k-th firm characteristic of stock i in month t - 1, and α^t and β_k^t are the corresponding regression intercepts and coefficients in month t. Then we take the time series average of α^t and β_k^t to get the final α and β_k , respectively. Their t-statistics are calculated in the same way as in Fama and MacBeth (1973). The firm characteristic in the

Table 4
Alphas for V/P Double-Sorted Portfolios: 1978–2018.

Panel A: Raw returns of 5–1 V/P long-short portfolio													
Quintile	ME	ME_M	B/M	B/M_M	OP	Inv	NS	Ac/B	Beta	Var	RVar	Mom	Turnover
1 (low)	0.69***	0.61***	0.1	0.18	0.68***	0.58**	0.48**	0.13	0.60***	0.27	0.33*	0.64***	0.54***
2	0.64***	0.65***	0.57***	0.33*	0.26	0.76***	0.68***	0.31	0.02	0.25	0.39**	0.59***	0.61***
3	0.62***	0.67***	0.31	0.50**	0.45*	0.27	0.43*	0.71***	0.09	0.54**	0.26	0.60***	0.18
4	0.55***	0.42*	0.40*	0.54**	0.71***	0.37*	−0.07	0.72***	0.63***	0.25	0.53**	0.46**	0.50**
5 (high)	0.33	0.33	0.41*	0.36	0.21	−0.03	0.41*	0.44**	0.65***	0.46*	0.50*	0.2	0.27

Panel B: Monthly CAPM alpha of 5–1 V/P long-short portfolio													
Quintile	ME	ME_M	B/M	B/M_M	OP	Inv	NS	Ac/B	Beta	Var	RVar	Mom	Turnover
1 (low)	0.76***	0.63***	0.32	0.40*	0.87***	0.63***	0.55***	0.24	0.69***	0.34*	0.50**	0.78***	0.68***
2	0.67***	0.70***	0.62***	0.36*	0.48**	0.86***	0.81***	0.41*	0.06	0.35*	0.53**	0.74***	0.79***
3	0.67***	0.70***	0.35*	0.54***	0.55**	0.41*	0.55**	0.87***	0.16	0.57**	0.34	0.73***	0.32
4	0.62***	0.50***	0.51**	0.65***	0.81***	0.46**	0.07	0.88***	0.64***	0.27	0.58**	0.56***	0.53**
5 (high)	0.52**	0.51**	0.56**	0.50**	0.34	0.12	0.54**	0.50**	0.59**	0.53*	0.54**	0.24	0.29

Panel C: Monthly AF3 alpha of 5–1 V/P long-short portfolio													
Quintile	ME	ME_M	B/M	B/M_M	OP	Inv	NS	Ac/B	Beta	Var	RVar	Mom	Turnover
1 (low)	0.68***	0.56***	0.23	0.33*	0.82***	0.63***	0.53***	0.14	0.62***	0.28*	0.43**	0.77***	0.61***
2	0.59***	0.62***	0.58***	0.39**	0.43**	0.82***	0.76***	0.35*	−0.02	0.28	0.46**	0.71***	0.76***
3	0.56***	0.59***	0.33*	0.56***	0.45**	0.35*	0.46**	0.81***	0.05	0.47**	0.26	0.72***	0.23
4	0.47***	0.37**	0.50**	0.65***	0.72***	0.38*	−0.09	0.81***	0.49**	0.16	0.47*	0.53***	0.41**
5 (high)	0.43**	0.43**	0.56**	0.53**	0.25	−0.01	0.43**	0.41**	0.47**	0.42	0.4	0.18	0.17

Panel D: Monthly HMXZ5 alpha of 5–1 V/P long-short portfolio													
Quintile	ME	ME_M	B/M	B/M_M	OP	Inv	NS	Ac/B	Beta	Var	RVar	Mom	Turnover
1 (low)	0.68***	0.44***	0.21	−0.02	0.68*	0.63**	0.32	0.08	0.44*	0.28	0.28	0.61*	0.54**
2	0.54***	0.55***	0.48**	0.32	0	0.67***	0.63**	0.25	−0.02	0.09	0.26	0.79**	0.50**
3	0.64***	0.68***	0.4	0.46*	0.39	0.18	0.73**	0.93***	0.13	0.38	0.16	0.38*	0
4	0.47*	0.52*	0.31	0.48*	0.72**	0.63*	0.19	0.74***	0.70**	0.2	0.2	0.19	0.39
5 (high)	0.25	0.21	0.16	0.15	0.12	0.13	0.35	0.24	0.51*	0.32	0.29	−0.25	0.12

Panel E: Monthly FF5 alpha of 5–1 V/P long-short portfolio													
Quintile	ME	ME_M	B/M	B/M_M	OP	Inv	NS	Ac/B	Beta	Var	RVar	Mom	Turnover
1 (low)	0.56***	0.37***	0.11	0.07	0.82***	0.72***	0.45**	0.05	0.64***	0.35**	0.32*	0.73***	0.65***
2	0.48***	0.47***	0.53**	0.40**	0.16	0.77***	0.73***	0.33	−0.01	0.19	0.31	0.77***	0.72***
3	0.43**	0.54***	0.40*	0.62***	0.36	0.32	0.62**	0.91***	0.02	0.31	0.16	0.74***	0.13
4	0.34*	0.3	0.49**	0.61***	0.71***	0.55**	−0.06	0.83***	0.48*	0.06	0.3	0.43**	0.39*
5 (high)	0.3	0.28	0.48*	0.57**	0.13	−0.08	0.43	0.3	0.38	0.33	0.24	−0.04	−0.01

Panel F: Monthly FF5+UMD alpha of 5–1 V/P long-short portfolio													
Quintile	ME	ME_M	B/M	B/M_M	OP	Inv	NS	Ac/B	Beta	Var	RVar	Mom	Turnover
1 (low)	0.70***	0.48***	0.29	0.21	0.92***	0.80***	0.51**	0.21	0.70***	0.43**	0.42**	0.78***	0.68***
2	0.60***	0.62***	0.66***	0.42**	0.2	0.89***	0.82***	0.4	0.14	0.26	0.4	0.79***	0.77***
3	0.62***	0.72***	0.49**	0.65***	0.48*	0.41*	0.77***	0.99***	0.19	0.39*	0.27	0.69***	0.26
4	0.53***	0.48**	0.53**	0.60**	0.82***	0.72***	0.16	0.94***	0.70**	0.26	0.43*	0.40**	0.55**
5 (high)	0.41*	0.38*	0.47*	0.53**	0.29	0.07	0.56**	0.44**	0.54**	0.47	0.44	−0.06	0.15

Panel G: Monthly AFP5 alpha of 5–1 V/P long-short portfolio													
Quintile	ME	ME_M	B/M	B/M_M	OP	Inv	NS	Ac/B	Beta	Var	RVar	Mom	Turnover
1 (low)	0.57***	0.37**	0.22	0.11	0.75**	0.56**	0.51**	0.2	0.56**	0.38**	0.40**	0.75**	0.81***
2	0.46***	0.47***	0.50**	0.32	0.1	0.85***	0.86***	0.35	0.14	0.34	0.48*	0.70**	0.79***
3	0.56***	0.69***	0.42*	0.59**	0.44*	0.42*	0.73**	0.91***	0.3	0.4	0.3	0.65***	0.1
4	0.46**	0.39*	0.46*	0.52**	0.82***	0.68**	0.18	0.90***	0.72***	0.23	0.3	0.40*	0.48**
5 (high)	0.35	0.32	0.28	0.39	0.27	0.06	0.45	0.35	0.44*	0.37	0.27	−0.05	0.16

Panel H: Monthly SY4 alpha of 5–1 V/P long-short portfolio													
Quintile	ME	ME_M	B/M	B/M_M	OP	Inv	NS	Ac/B	Beta	Var	RVar	Mom	Turnover
1 (low)	0.79***	0.59***	0.39	0.28	1.02***	0.77***	0.62***	0.36	0.60***	0.47**	0.53***	0.92***	0.63***
2	0.72***	0.76***	0.79***	0.50**	0.25	0.97***	0.89***	0.53*	0.26	0.34	0.46*	0.77***	0.84***
3	0.86***	0.88***	0.64**	0.67***	0.58**	0.50**	0.89***	1.04***	0.34	0.58**	0.38	0.65***	0.3
4	0.62***	0.60**	0.59**	0.58**	0.81***	0.84**	0.44	1.00***	0.83***	0.52	0.5	0.39*	0.59**
5 (high)	0.47*	0.44*	0.49*	0.59**	0.36	0.18	0.49	0.33	0.4	0.58	0.68**	−0.02	0.37

Panel I: Monthly BS6 alpha of 5–1 V/P long-short portfolio													
Quintile	ME	ME_M	B/M	B/M_M	OP	Inv	NS	Ac/B	Beta	Var	RVar	Mom	Turnover
1 (low)	0.55***	0.31**	0.23	0.2	0.75***	0.74***	0.42**	0.08	0.52**	0.25	0.26	0.68**	0.56***
2	0.40***	0.37***	0.58**	0.38*	−0.03	0.81***	0.69***	0.18	0.02	0.13	0.26	0.62**	0.64***
3	0.40**	0.53***	0.34	0.48**	0.19	0.23	0.62**	0.78***	0.08	0.2	0.1	0.50**	0.02
4	0.26	0.23	0.31	0.45*	0.64**	0.59**	−0.12	0.82***	0.41	0.16	0.16	0.3	0.39*
5 (high)	0.23	0.21	0.2	0.28	0.19	−0.12	0.27	0.36	0.43*	0.21	0.17	−0.26	−0.05

This table presents the raw percentage returns (Panel A) and the intercepts (alphas in percentage terms in Panel B-I) for the time series regressions of monthly return of 5 (High) minus 1 (Low) V/P portfolios within each firm characteristic quintile on different factors during July 1978–June 2018: $R_{HL,i}^t = \alpha_i + \sum_{k=1}^K \beta_{i,k} f_k^t + \epsilon_i^t$, where $R_{HL,i}^t$ is the month t return difference between the highest and lowest V/P quintile (5–1 long-short portfolio) within a firm characteristic quintile i, and f_k^t is the value of kth factor in month t (monthly return for traded factors) in a factor model. Each portfolio is constructed as follows: at the end of each month, we do a 5×5 double sort on V/P and each firm characteristic. Stocks are dependently split into five V/P ratios according to the ranking of B/M, B/M_M and Mom, or independently split by the ranking of other firm characteristics in the Table 4. V/P quintile 1 (5) includes the 20% stocks with the lowest (highest) V/P ratios, and a long-short portfolio V/P 5–1 that buys stocks in V/P quintile 5 and shorts stocks in V/P quintile 1 is also constructed within each of the 5 firm characteristic quintiles at the same time. Each long-short portfolio is then held for 1 month, all returns are value-weighted. The factor models include: basic CAPM, AF3 (Asness and Frazzini (2013) 3-factor model), HMXZ5 (Hou et al. (2021) q5-factor model), FF5 (Fama and French (2015) 5-factor model), FF5+UMD (Fama and French (2015) 5-factor plus Momentum factor model), AFP5 (Carhart (1997) 4-factor plus Asness et al. (2019) Quality-Minus-Junk factor model), SY4 (Stambaugh and Yuan (2016) 4-factor mispricing model), and BS6 (Barillas and Shanken (2018) 6-factor model). The definitions for ME, ME_M, B/M, B/M_M, OP, Inv, NS, Ac/B, Beta, Var, RVar, Mom, and Turnover are listed in the appendix. *, **, *** indicate significance at 10%, 5%, 1% level, respectively. All t-statistics are White (1980) t-statistics.

Table 5
Fama-MacBeth Regression: 1978–2018.

Panel A		Adj. R-squared	Int	V/P	ME	B/M_M	OP Neg	OP Pos	Inv	
Average	0.07***	1.18***	0.94***	-0.00***	0.06	0.18**	0.13	-0.37**		
t-statistic	25.38	5.94	6.59	-2.61	0.72	2.2	0.45	-2.1		
Panel B		NS	NS Zero	Ac/B Neg	Ac/B Pos	Beta	Var	RVar	Mom	Turnover
Average	-1.27***	0.01	-0.43	-0.18	0.16**	-0.74	3.40	0.22***	-0.41**	
t-statistic	-3.01	0.28	-0.94	-0.41	2.04	-0.26	1.33	6.65	-2.5	

The table shows the time series average and t-statistics of the intercepts and slopes of 480 cross-sectional regression of stock *i*'s month *t* return on its various firm characteristics at month *t*-1 during July 1978-June 2018: $R_i^t = \alpha^t + \sum_{k=1}^K \beta_k^t X_{i,k}^{t-1} + \epsilon_i^t$, where R_i^t is monthly return of stock *i* in month *t*, $X_{i,k}^{t-1}$ is the *k*th firm characteristic of stock *i* in month *t*-1. α^t and β_k^t are the corresponding regression intercepts and coefficients in month *t*. The firm characteristic in the regressions include V/P, ME, B/M_M, OP Neg (dummy for negative OP), OP Pos (dummy for positive OP), Inv, NS, NS Zero (dummy for zero NS), Ac/B Neg (dummy for negative Ac/B), Ac/B Pos (dummy for positive Ac/B), Beta, Var, RVar, Mom, and Turnover. V/P is the fundamental value *V* calculated on the month-end using a 3-period Residual Income Model (Eq. (2)) divided by the market cap on the month-end. The definitions for ME, B/M, B/M_M, OP, Inv, NS, Ac/B, Beta, Var, RVar, Mom, and Turnover are listed in the appendix. In particular, OP Neg (Ac/B Neg) is one if OP (Ac/B) is negative and zero otherwise, while OP Pos (Ac/B Pos) is OP (Ac/B) if OP (Ac/B) is positive and zero otherwise. NS Zero is one if NS is zero and zero otherwise; Standard errors are baseline Fama and Macbeth (1973) standard errors, *, **, *** indicate significance at 10%, 5%, 1% level, respectively.

regressions include V/P, ME, B/M_M, OP Neg (dummy for negative OP), OP Pos (OP if it is positive and zero otherwise), Inv, NS, NS Zero (dummy for zero NS), Ac/B Neg (dummy for negative Ac/B) and Ac/B Pos (Ac/B if it is positive and zero otherwise), Beta, Var, RVar, Mom, and Turnover.

Table 5 presents the time series average coefficients and their *t*-statistics from the Fama-MacBeth regression (Eq. (3)). We find that the V/P ratio has the highest *t*-statistic (6.59) with a slope of 0.94. Even though the V/P ratio is correlated with the book-to-market ratio (B/M_M), B/M_M has no predictive power when the V/P ratio is simultaneously included, as indicated by a *t*-statistic of only 0.72. Conversely, the V/P ratio does have predictive power by construction since it incorporates analysts' forecasts. Furthermore, we find that the other supposed predictive measures, such as investment (Inv) and profitability (OP), are much less predictive than the V/P ratio when we control for the other variables.

4. Value-price divergence as a factor

Next, we construct the value-price-divergence (VPD) factor inspired by the RIM-based premium: each month, stocks are independently sorted into three V/P portfolios and two size portfolios. The three V/P portfolios are: the underpriced (bottom 30% V/P), the neutral (middle 40% V/P), and the overpriced (top 30% V/P) stocks. The two size portfolios are the small (bottom 50%) and the big (top 50%) stocks. The VPD factor's return is the average of the return difference between the underpriced and the overpriced stocks across small and big groups of stocks.

In SubSection 4.1, we discuss the excess returns of the VPD factor portfolio, as well as how the VPD factor relates to other factors. In SubSection 4.2, we interpret the VPD factor.

4.1. The VPD factor

To investigate whether the VPD factor differs from the monthly HML factor. For robustness, we also compare VPD to HMLM (the monthly HML devil factor of Asness and Frazzini, 2013). We calculate the monthly returns of the VPD factor from July 1978 to June 2018, and then compare them with those of the monthly HML devil factor, market risk premium (MKT in FF3 or FF5 model), size (SMB in FF3/5 model and ME factor in HMXZ5 model), value (HML in FF3/5 model), investment (CMA in FF5 and IA in HMXZ5), profitability (RMW in FF5 and ROE in HMXZ5 model), growth (EG in

HMXZ5 model), momentum (UMD in Carhart model), and quality-minus-junk (QMJ) factors. We also consider the monthly returns of the MGMT factor (the first cluster factor in SY4 model) and the PERF factor (the second cluster factor in SY4 model).

Table 6 presents the summary statistics of annualized monthly returns for all factors considered. We find that the VPD factor has an average annual return of 5.22% and an annual Sharpe ratio of 0.57, both of which are significantly different from the annual return of 2.28% and Sharpe ratio of 0.19 for the monthly HML factor. This confirms that the VPD factor performs very differently from the monthly HML factor, and its 5.22% annual return is economically significant and higher than that of the size, value, and investment factors. We also find that the risk and return profile of the VPD factor is similar to that of the profitability factor (RMW) in the FF5 model, with the latter having a slightly lower return (4.25%) and Sharpe ratio (0.52). Among all factors, the market, momentum, EG, MGMT, and PERF factors yield the highest annual return (above 7%), accompanied by higher annual volatility (over 15%) in the case of market and momentum. In addition, the investment and profitability factors in the HMXZ5 model have higher Sharpe ratios than those in the FF5 model, while the growth factor EG in the HMXZ5 model has the highest Sharpe ratio (1.42) among all factors.

Fig. 3 presents the cumulative return for the momentum and value related factors (i.e., VPD, HMLM, UMD, IA, ROE, CMA, RMW, and QMJ) since the June-end of 1978 to the June-end of 2018. We find that the VPD, profitability, and investment factors have similar patterns of growth earlier in the sample period. However, since 2000, the VPD factor significantly outperforms the other two factors, only being edged out by the QMJ factor at the end of the sample period. The momentum and ROE factors have the highest returns but come with the highest volatility and drawdown.

Table 7 shows that the VPD factor is most correlated with the monthly HML factor (HMLM) with a correlation of 0.48, and it is second most and positively correlated with the profitability factor RMW and the HML, displaying a correlation of 0.42 with both. Return on equity (ROE) can be considered as a measure of profitability, and it is an input of the residual income model. Furthermore, VPD is negatively correlated with MKT, SMB, ME, UMD, and PERF, and their correlations are -0.28, -0.34, -0.26, -0.33, and -0.09, respectively. The correlations between VPD and IA, CMA, EG, QMJ are all near 0.25. Consistent with the q-theory, we also find that the investment factors (CMA and IA) are positively correlated with

Table 6
Summary Statistics of Monthly Factor Returns: 1978–2018.

	VPD	HMLM	MKT	SMB3	HML	UMD	ME	IA	ROE	EG	CMA	RMW	QMJ	MGMT	PERF
# Observations	480	480	480	480	480	480	480	480	480	480	480	480	480	462	462
Mean	0.43	0.19	0.67	0.15	0.25	0.62	0.22	0.32	0.56	0.77	0.26	0.35	0.46	0.59	0.70
Std. Dev.	2.65	3.53	4.40	3.02	2.90	4.45	2.97	1.88	2.55	1.88	1.98	2.35	2.37	2.86	4.04
Min	-13.06	-17.99	-23.24	-16.87	-11.10	-34.39	-14.39	-7.15	-13.85	-6.29	-6.88	-18.37	-9.10	-8.93	-21.45
25%	-0.99	-1.66	-1.92	-1.57	-1.37	-1.03	-1.46	-0.88	-0.69	-0.40	-0.99	-0.82	-0.80	-1.15	-1.50
50%	0.24	-0.03	1.09	0.09	0.03	0.70	0.19	0.30	0.65	0.66	0.13	0.32	0.46	0.55	0.62
75%	1.80	1.81	3.51	1.87	1.70	2.91	1.96	1.40	1.85	1.84	1.48	1.35	1.60	2.22	2.76
Max	11.55	26.86	12.47	21.71	12.90	18.36	22.14	9.25	10.38	10.93	9.58	13.31	12.39	14.58	18.52
Annualized Mean	5.22	2.28	8.02	1.81	2.98	7.39	2.70	3.80	6.76	9.23	3.11	4.25	5.55	7.06	8.37
Annualized Std.	9.18	12.23	15.24	10.45	10.04	15.43	10.29	6.50	8.85	6.51	6.87	8.13	8.20	9.90	13.99
Annualized Sharpe	0.57	0.19	0.53	0.17	0.30	0.48	0.26	0.58	0.76	1.42	0.45	0.52	0.68	0.71	0.60

This table presents the summary statistics of various factors' monthly returns (percent) during July 1978–June 2018. The *VPD* factor is constructed as follows: each month, stocks are independently sorted into 3 *V/P* portfolios and 2 size portfolios. The three *V/P* portfolios are underpriced (top 30% *V/P*), neutral (middle 40% *V/P*), and overpriced (bottom 30% *V/P*) stocks, and the two size portfolios are small (bottom 50%) and big (top 50%) stocks. Then the *VPD* factor's return is the average of the return difference of the underpriced and overpriced stocks within small and big groups of stocks. *HMLM* is the monthly *HML* devil factor of [Asness and Frazzini \(2013\)](#) 3-factor model and is downloaded from AQR's website. The monthly returns of *MKT*, *SMB3*, and *HML* of the [Fama and French \(1993\)](#) 3-factor model, *CMA*, and *RMW* of the [Fama and French \(2015\)](#) 5-factor model, *UMD* (momentum) of the [Carhart \(1997\)](#) model are downloaded from Ken French's Data Library. The monthly returns of *ME*, *IA*, *ROE*, and *EG* (size, investment, profitability, and expected growth factors in [Hou et al. \(2021\)](#) 5-factor model) are downloaded from global-q.org. The monthly returns of *QMJ* ([Asness-Frazzini-Pedersen \(2019\)](#) quality minus junk factor) are downloaded from AQR's website. The monthly returns of *MGMT* and *PERF* (the 1st and 2nd cluster factor in [Stambaugh and Yuan \(2016\)](#) 4-factor model) are downloaded from Robert F. Stambaugh's home page.

Table 7
Correlation Table: 1978–2018.

	VPD	HMLM	MKT	SMB3	HML	UMD	ME	IA	ROE	EG	CMA	RMW	QMJ	MGMT	PERF
VPD	1														
HMLM	0.48	1													
MKT	-0.28	-0.12	1												
SMB3	-0.34	-0.19	0.24	1											
HML	0.42	0.76	-0.28	-0.26	1										
UMD	-0.33	-0.7	-0.1	0.08	-0.22	1									
ME	-0.26	-0.1	0.24	0.95	-0.1	0.08	1								
IA	0.23	0.5	-0.36	-0.23	0.69	-0.01	-0.12	1							
ROE	0.2	-0.37	-0.23	-0.36	-0.01	0.51	-0.27	0.09	1						
EG	0.25	-0.08	-0.41	-0.39	0.17	0.3	-0.34	0.29	0.55	1					
CMA	0.25	0.49	-0.39	-0.14	0.69	-0.02	-0.05	0.91	-0.01	0.25	1				
RMW	0.42	0.04	-0.31	-0.48	0.22	0.11	-0.4	0.21	0.71	0.54	0.12	1			
QMJ	0.26	-0.21	-0.55	-0.45	0.03	0.28	-0.42	0.16	0.7	0.65	0.12	0.75	1		
MGMT	0.36	0.48	-0.51	-0.4	0.72	0	-0.31	0.76	0.16	0.5	0.77	0.35	0.41	1	
PERF	-0.09	-0.63	-0.28	-0.08	-0.29	0.73	-0.09	-0.04	0.65	0.5	-0.02	0.43	0.64	0.03	1

This table presents the correlations among various factors' monthly returns during the sample period July 1978–June 2018. The *VPD* factor is defined as in Section 4 and Table 6. *HMLM* is the monthly *HML* devil factor of *Asness and Frazzini (2013)* 3-factor model and is downloaded from AQR's website, the monthly returns of *MKT*, *SMB3*, and *HML* of the *Fama and French (1993)* 3-factor model, *CMA*, and *RMW* of the *Fama and French (2015)* 5-factor model, *UMD* (momentum) of the *Carhart (1997)* model are downloaded from Ken French's Data Library. The monthly returns of *ME*, *IA*, *ROE*, and *EG* (size, investment, profitability, and expected growth factors in *Hou et al. (2021)* q5-factor model) are downloaded from global-q.org. The monthly returns of *QMJ* (*Asness-Frazzini-Pedersen (2019)* quality minus junk factor) are downloaded from AQR's website. The monthly returns of *MGMT* and *PERF* (the 1st and 2nd cluster factor in *Stambaugh and Yuan (2016)* 4-factor model) are downloaded from Robert F. Stambaugh's home page.

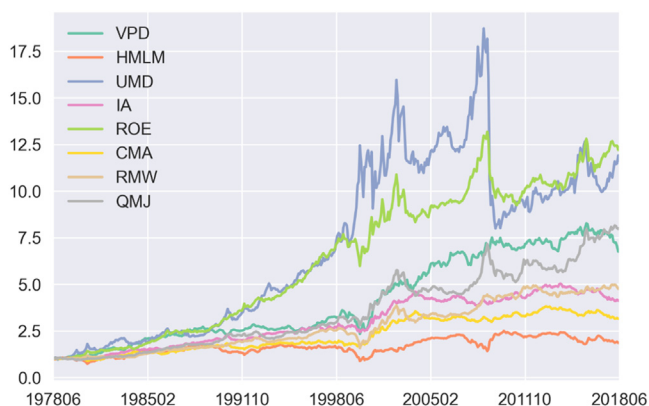


Fig. 3. Cumulative Returns of Factors: 1978–2018. This figure presents various factors' cumulative returns during the sample period from June 1978 to June 2018. The *VPD* factor is constructed as follows: each month, stocks are independently sorted into 3 *V/P* portfolios and 2 size portfolios. The three *V/P* portfolios are underpriced (top 30% *V/P*), neutral (middle 40% *V/P*), and overpriced (bottom 30% *V/P*) stocks, and the two size portfolios are small (bottom 50%) and big (top 50%) stocks. Then the *VPD* factor's return is the average of the return difference of the underpriced and overpriced stocks within small and big groups of stocks. *HMLM* is the monthly *HML* devil factor of *Asness and Frazzini (2013)* 3-factor model and is downloaded from AQR's website, *CMA*, and *RMW* of the *Fama and French (2015)* 5-factor model, *UMD* (momentum) of the *Carhart (1997)* model are downloaded from Ken French's Data Library. The monthly returns of *IA*, *ROE* (investment, profitability factors in *Hou et al. (2021)* q5-factor model) are downloaded from global-q.org. The monthly returns of *QMJ* (*Asness-Frazzini-Pedersen (2019)* quality minus junk factor) are downloaded from AQR's website.

the value factor *HML*. Intuitively, according to the first principle of investment, the marginal costs of investment, which rise with investment, equal marginal *q*, which is closely related to book-to-market equity.

Next, we run regressions of the *VPD* factor on these competing factors to see if its return could be explained by some of these factors (Table 8). We start from the CAPM, and then sequentially add more factors into the regressions. In all cases, we find a significant monthly alpha ranging from 0.26% to 0.55%, indicating that these factors do not fully explain the *VPD* factor's return. In addition, the *VPD* factor has positive loadings on the value, profitability, and quality-minus-junk factors, and negative loadings on the momentum, size, and market factors; however, the loadings on size are

not significant when *CMA* and *RMW* are added into the model. We also emphasize that when the traditional annual value factor *HML* is replaced with monthly value factor *HMLM*, the alpha is still positive and significant, although its magnitude decreases. Thus, the *VPD* factor is not spanned by *HMLM*.

4.2. Interpretations of *VPD*

VPD as a risk factor can be interpreted similarly to how we usually interpret the value factor. For example, cheap firms (high *V/P* ratio) tend to exhibit fewer stable earnings and higher debt levels for which investors demand compensation in the form of higher returns (e.g., *Fama and French, 1993*). *VPD* could also be a mispricing from investors' behavioral patterns: investors tend to shun stocks that have recently underperformed (e.g., *Jegadeesh and Titman, 2001*), and thus are likely have a low *P*-value for a given *V*-value. This potential gap is only corrected over time through various limits to arbitrage (e.g., *Frankel and Lee, 1998*). The demand/attention drop leads to an excessive price drop that would revert over time, which yields a higher return. A pricing factor constructed from such temporary behavioral mispricing in the market can then be viewed as a statistical, systematic factor, not a rational risk factor, exposure to which demands a premium.

In reality, *VPD* likely involves a combination of rational and behavioral elements, as *Shiller (1984)* argues, and as also discussed in the noise-trading approach to finance (e.g., *Shleifer and Summers, 1990; Campbell and Kyle, 1993*). *VPD* (and the price premium it commands), cast in *Shiller's (1984)* model, can be viewed as a measure of systematic market noise or the severity of the limits to arbitrage. To see this, if a deviation of market price from the estimate of the asset's intrinsic value leads to a high expected return on average, then some significant mispricing is not arbitrated away immediately, and may reflect noise trading in the market or severe limits to arbitrage. If a firm's return is positively correlated with the *VPD* factor, then fundamental investors may view it as risky because its return is more likely driven by market noise than by the intrinsic values. In the framework of *Shiller (1984)*, the *VPD* premium is therefore an indicator of market noise. A high correlation with the *VPD* factor means a firm is more exposed to market noise, which is risky.

Regardless of the interpretation, *VPD* clearly substitutes for the conventional value factor and, as we show later, the four-factor

Table 8
Regression of the VPD Factor on Other Factors: 1978–2018.

Regressors/Model	Constant	CAPM	FF3	AF3	HMXZ5	FF5	FF5_M	FF5+UMD	FF5+UMD_M	AFP5	AFP5_M	SY4	BS6
ALPHA	0.43***	0.55***	0.45***	0.47***	0.37**	0.33***	0.32***	0.44***	0.36***	0.39***	0.26**	0.54***	0.29***
MKT		-0.17***	-0.08**	-0.10***	-0.09**	-0.06*	-0.08**	-0.08**	-0.08**	-0.03	-0.02	-0.10***	-0.10***
SMB			-0.20***	-0.19***		-0.09	-0.07	-0.06	-0.06	-0.09*	-0.08	-0.15**	-0.03
HML			0.29***			0.35***		0.22***		0.27***			
HMLM				0.31***			0.37***		0.28***		0.34***		0.46***
CMA						-0.13	-0.12	-0.03	-0.06				
RMW						0.31***	0.38***	0.38***	0.40***				
UMD								-0.20***	-0.07*	-0.20***	-0.06		-0.10**
ME						-0.16**							
IA						0.19*							-0.28***
ROE						0.09							0.50***
EG						0.06							
QMJ										0.31***	0.37***		
MGMT												0.22***	
PERF												-0.10***	

This table presents the coefficients of the time series regressions of VPD factor's monthly return on a constant one and various factors: $R_{VPD}^t - R_f^t = \alpha + \sum_{k=1}^K \beta_k f_k^t + \epsilon^t$, where R_{VPD}^t and R_f^t is the month t return of the VPD factor and the risk-free asset, respectively, and f_k^t is the value of kth factor in month t (monthly return for traded factors) in a factor model. The following models are included in the regressions: CAPM, FF3 ((Fama and French, 1993) 3-factor model), AF3 (Asness and Frazzini (2013) 3-factor model), HMXZ5 (Hou et al. (2021) q5-factor model), FF5 (Fama and French (2015) 5-factor model), FF5_M (FF5 model with HML replaced by HMLM), FF5+UMD (Fama and French (2015) 5-factor plus Momentum factor model), FF5+UMD_M (FF5+UMD with HML replaced by HMLM), AFP5 (Carhart (1997) 4-factor plus Asness et al. (2019) Quality-Minus-Junk factor model), AFP5_M (AFP5_M with HML replaced by HMLM), SY4 (Stambaugh and Yuan (2016) 4-factor mispricing model), and BS6 (Barillas and Shanken (2018) 6-factor model); The VPD factor is defined as in Section 4 and Table 6. HMLM is the monthly devil HML factor of Asness and Frazzini (2013) 3-factor and is downloaded from AQR's website. The monthly returns of MKT, SMB3, and HML of the Fama and French (1993) 3-factor model, CMA, and RMW of the Fama and French (2015) 5-factor model, UMD (momentum) of the Carhart (1997) model are downloaded from Ken French's Data Library. The monthly returns of ME, IA, ROE, and EG (size, investment, profitability, and expected growth factors in Hou et al. (2021) 5-factor model) are downloaded from global-q.org. The monthly returns of QMJ (Asness-Frazzini-Pedersen (2019) quality minus junk factor) are downloaded from AQR's website. The monthly returns of MGMT and PERF (the 1st and 2nd cluster factor in Stambaugh and Yuan (2016) 4-factor model) are downloaded from Robert F. Stambaugh's home page. The intercepts (ALPHA) of regressions are transformed into percentage terms. *, **, *** indicate significance at 10%, 5%, 1% level, respectively. All t-statistics are White (1980) t-statistics.

model built on *VPD* prices the cross-section even better than the more recent [Fama and French \(2015\)](#) 5-factor model. Though RIM valuation is not perfect, it is forward-looking and incorporates the forecasts by analysts who consider real options and intangibles of a firm with broad implications for value investment professionals and asset pricers .

5. Model comparisons

A GRS test ([Gibbons et al., 1989](#)) is commonly used to jointly test the significance of alphas of the regressions of portfolio excess returns on model factors, and thus the explanatory power of the models. The limitation of the GRS test is that the inferences can vary across sets of portfolios. However, if a model really explains asset returns well, it should not matter which portfolios are chosen.

In this section, we use two more robust methods to compare models. In Subsection 5.1, we employ the max squared Sharpe ratio test ([Barillas and Shanken, 2017](#)) to compare *HML* related models before and after *HML* is replaced by *VPD*. In Subsection 5.2, we implement the constrained R-squared method proposed by [Maio \(2019\)](#) to compare all factor models, including those without the *HML* factor.

5.1. Max squared sharpe ratio test

[Barillas and Shanken \(2017\)](#) argue that models should be compared in terms of their ability to price all returns that include both test assets and factors. Thus, when comparing different asset pricing models, the test assets should be augmented by factors from other models. Because test assets provide no additional information beyond what we learn by examining how well each model prices the factors in the other models, [Barillas and Shanken \(2017\)](#) propose that we judge each model by the maximum (max) squared Sharpe ratio that can be constructed with the intercepts from time series regressions of excess returns on all assets on the model's factors. Specifically, we denote Π as excess returns for all assets, f as a model's factors, α as the vector of intercepts from the regressions of Π on f , and Σ as the residual covariance matrix. The max squared Sharpe ratio for the intercepts is $Sh^2(\alpha) = \alpha' \Sigma^{-1} \alpha$, and the best model is the one with the smallest $Sh^2(\alpha)$. ([Gibbons et al., 1989](#)) show that $\alpha' \Sigma^{-1} \alpha = Sh^2(\Pi, f) - Sh^2(f)$, and since the set of all asset returns Π includes f , $Sh^2(\Pi, f) = Sh^2(\Pi)$. Therefore, the best model is the one whose factors have the highest max squared Sharpe ratio, $Sh^2(f)$, and the intercepts for any subset of left-hand-side (LHS) assets add nothing to the information in $Sh^2(f)$. The max squared Sharpe ratio of a factor model, $Sh^2(f)$, can be calculated as $\mu_f' V_f^{-1} \mu_f$, where μ_f is the vector of mean factor returns, and V_f is the variance-covariance matrix for the vector of factor returns.

We use this insight to compare the six base and six alternative models. Specifically, we compare the following six base models with alternative models replacing *HML* with *VPD* factor:

1. AF3 model: $MKT + SMB + HMLM$,
2. Carhart model: $MKT + SMB + HML + UMD$,
3. FF5 model: $MKT + SMB + HML + CMA + RMW$,
4. FF5+UMD model: $MKT + SMB + HML + CMA + RMW + UMD$,
5. AFP5 model: $MKT + SMB + HML + UMD + QMJ$.
6. BS6 model: $MKT + SMB + HMLM + IA + ROE + UMD$

[Table 9](#) shows that replacing the *HML* or *HMLM* factor with the *VPD* factor increases the max squared Sharpe ratio for all six base models. The highest improvement is seen for the Carhart model, where the max squared Sharpe ratio increases from 8% to 14%. Furthermore, the max squared Sharpe ratio of the

$MKT + SMB + VPD + UMD$ model is higher than that of the FF5 model, indicating that *VPD* and *UMD* together may potentially explain the *HML*, *CMA*, and *RMW* factors. Adding more factors into the model also increases the max squared Sharpe ratio. The highest max squared Sharpe ratio of 22% comes from the AFP5 related models, where replacing *HML* with *VPD* yields a similar max squared Sharpe ratio; the AFP5 and BS6 models have similar max squared Sharpe ratios ranging from of 20% to 22%. To see if the improvement in the max squared Sharpe ratio when *HML* or *HMLM* is replaced with *VPD* is statistically significant, we calculate the 90% confidence interval of the difference between the two max squared Sharpe ratios using 10,000 bootstraps. The results show that replacing *HML* with *VPD* significantly increases the max squared Sharpe ratio of the AF3, Carhart, and FF5 + *UMD* factor models, but not for the FF5, AFP5, or BS6 models.

Panel B of [Table 9](#) shows that regressing *VPD* on the other factors in the model leaves a positively significant alpha in all six regressions. The alphas range from 0.3% to 0.69% and all are statistically significant at a 95% level; five out of six are statistically significant at a 99% level. On the other hand, even though regressing the *HML* factor on the other factors in either the FF3 or Carhart model generates significant alphas, regressing it on the other factors in models that include the *CMA* and *RMW* factors yields insignificant alphas. This confirms that the value factor becomes redundant when we include the profitability and investment factors in the model.

[Table 10](#) shows the marginal contribution of each factor to the max squared Sharpe ratio of the factor model. The marginal contribution of a factor to the max squared Sharpe ratio is the square of the ratio of the intercept in the spanning regression of the factor on the model's other factors to the standard deviation of the regression residuals. In all models, *VPD*'s marginal contribution to the model's max squared Sharpe ratio is significantly higher than that of the *HML* or *HMLM* factor. For example, *HML*'s marginal contribution to the Carhart model is only 3.31%; however, *VPD*'s marginal contribution to the alternative Carhart model that replaces *HML* with *VPD* is 9.36%. Furthermore, when *VPD* and *UMD* are jointly added into the model, they both contribute a substantial amount to the max squared Sharpe ratio. In the alternative Carhart model, the *VPD* and *UMD* factors contribute 9.36% and 6.52% to the max squared Sharpe ratio, respectively. Additionally, their marginal contributions are much higher than those of the *MKT* and *SMB* factors. In the alternative FF5 + *UMD* model, *VPD* and *UMD* contribute 4.41% and 4.73% to the max squared Sharpe ratio, respectively, which is less than the market factor (*MKT*), but more than the *RMW* and *CMA* factors. Consistent with the redundancy of *HML* in the FF 5-factor model, we find that the marginal contribution of *HML* in the FF 5-factor model is zero. Lastly, the marginal contribution of *VPD* in the alternative AFP5 model is 6.35%, the same as that of *HML* to the corresponding base model, and lower than that of the *QMJ* factor (8.21%) and market factor (11.79%). But the marginal contribution of the *QMJ* factor decreases from 14.25% in the base model to 8.21% in the alternative model, suggesting that some of the contribution is absorbed by the *VPD* factor.

In summary, the max squared Sharpe ratio test results confirm that replacing *HML* or *HMLM* with *VPD* substantially improves the max squared Sharpe ratio of six base models (i.e., the explanatory power of the model) and the *VPD* factor's marginal contribution to the max squared Sharpe ratio are significant.

5.2. Constrained R-squared test

[Cooper and Maio \(2019a, 2019b\)](#), [Cooper et al. \(2021\)](#), and [Maio \(2019\)](#) propose a new goodness-of-fit measure, the "constrained" cross-sectional R-squared, to evaluate factor models where all factors are excess stock returns. This new measure

Table 9
Comparison of Max Squared Sharpe Ratios: 1978–2018.

Panel A: Comparison of Max Squared Sharpe Ratios Model						
Model	Sh2(HML)	Sh2(VPD)	Sh2(VPD) – Sh2(HML)	% of >0	5%	95%
AF3: MKT+SMB+HMLM/VPD	0.0291	0.0754	0.0463	99.94	0.0184	0.0845
Carhart: MKT+SMB+HML/VPD+UMD	0.0801	0.1406	0.0605	97.26	0.0087	0.1199
FF5: MKT+SMB+HML/VPD+CMA+RMW	0.1198	0.1349	0.0151	85.77	-0.0075	0.0464
FF5+UMD: MKT+SMB+HML/VPD+CMA+RMW+UMD	0.1382	0.1822	0.0440	99.41	0.0115	0.0884
AFP5: MKT+SMB+HML/VPD+UMD+QMJ	0.2226	0.2227	0.0001	48.43	-0.0598	0.057
BS6: MKT+SMB+HMLM/VPD+IA+ROE+UMD	0.2034	0.2111	0.0077	62.46	-0.0326	0.05

Panel B: Regression of HML/HMLM/VPD on the other Factors in the Model				
Model	Int (HML or HMLM)	Int (VPD)	t-statistic (HML or HMLM)	t-statistic (VPD)
AF3: MKT+SMB+HMLM/VPD	0.0026	0.0056	1.7052	4.8701
Carhart: MKT+SMB+HML/VPD+UMD	0.0048	0.0069	3.5553	6.1605
FF5: MKT+SMB+HML/VPD+CMA+RMW	-0.0008	0.003	-0.7189	2.3289
FF5+UMD: MKT+SMB+HML/VPD+CMA+RMW+UMD	0.0001	0.0044	0.1116	3.851
AFP5: MKT+SMB+HML/VPD+UMD+QMJ	0.0065	0.0056	4.9363	4.761
BS6: MKT+SMB+HMLM/VPD+IA+ROE+UMD	0.0035	0.0045	3.6057	3.795

Panel A presents max squared Sharpe ratios for six base & alternative models and their differences. The 5th, 95th percentiles of the difference and the fraction of positive difference are also calculated through 10,000 bootstraps. The maximum squared Sharpe ratio of a factor model is calculated as $\mu_f' V_f^{-1} \mu_f$, in which μ_f is the vector of mean factor returns, and V_f is the variance-covariance matrix for the vector of factor returns. Panel B reports intercepts and their t-statistics of time series regressions of HML, HMLM, or VPD factor on the other factors in the model. E.g., $HML_t = MKT_t + SMB_t + UMD_t + \epsilon_t$ for the Carhart (1997) 4-factor model. The six base models include AF3 (Asness and Frazzini (2013) 3-factor), Carhart (1997) 4-factor, Fama and French (2015) 5-factor, FF 5-factor plus momentum factor, AFP5 (Carhart (1997) 4-factor plus Asness et al. (2019) Quality-minus-Junk factor), and BS6 (Barillas and Shanken (2018) 6-factor) models. The alternative models are the base models with HML or HMLM replaced with VPD. The VPD factor is defined as in Section 4 and Table 6. HMLM is the monthly HML devil factor of Asness and Frazzini (2013) 3-factor, and is downloaded from AQR's website, the monthly returns of MKT, SMB, and HML of the Fama and French (1993) 3-factor model, CMA, and RMW of the Fama and French (2015) 5-factor model, UMD (momentum) of the Carhart (1997) model are downloaded from Ken French's Data Library. The monthly returns of IA and ROE (investment, profitability factors in Hou et al. (2021) 5-factor model) are downloaded from global-q.org. The monthly returns of QMJ (Asness-Frazzini-Pedersen (2019) quality minus junk factor) are downloaded from AQR's website. The sample period is from July 1978 to June 2018. All t-statistics are White (1980) t-statistics.

Table 10
Max Squared Sharpe Ratios and Factor Marginal Contributions: 1978–2018.

	Sh2(f)	MKT	SMB	HML/VPD	UMD	CMA	RMW	QMJ	IA	ROE
AF3: MKT+SMB+HMLM	0.0291	2.24%	0.07%	0.58%						
MKT+SMB+VPD	0.0754	3.57%	0.65%	5.22%						
Carhart: MKT+SMB+HML+UMD	0.0801	4.16%	0.10%	3.31%	3.76%					
MKT+SMB+VPD+UMD	0.1406	5.68%	0.69%	9.36%	6.52%					
FF5: MKT+SMB+HML+CMA+RMW	0.1198	6.62%	0.92%	0.14%	-	3.12%	5.43%			
MKT+SMB+VPD+CMA+RMW	0.1349	7.06%	1.30%	1.66%	-	3.17%	3.19%			
FF5+UMD: MKT+SMB+HML+CMA+RMW+UMD	0.1382	7.22%	0.69%	0.00%	1.84%	2.32%	4.23%			
MKT+SMB+VPD+CMA+RMW+UMD	0.1822	8.74%	1.01%	4.41%	4.73%	3.11%	1.30%			
AFP5: MKT+SMB+HML+UMD+QMJ	0.2226	13.86%	4.04%	6.34%	0.57%			14.25%		
MKT+SMB+VPD+UMD+QMJ	0.2227	11.79%	3.48%	6.35%	1.69%			8.21%		
BS6: MKT+SMB+HMLM+IA+ROE+UMD	0.2034	7.06%	2.62%	3.72%	1.81%				0.43%	7.57%
MKT+SMB+VPD+IA+ROE+UMD	0.2111	8.74%	2.44%	4.49%	1.04%				5.21%	2.25%

This table presents the max squared Sharpe ratios of twelve competing models and marginal contributions of each factor to the max squared Sharpe ratios of those models during July 1978–June 2018. The marginal contribution of a factor to the max squared Sharpe ratio of a factor model is the square of the ratio of the intercept in the spanning regression of the factor on the model's other factors to the standard deviation of the regression residuals. Each column of the table shows the max squared Sharpe ratio of the model's factors $Sh^2(f)$, and the marginal contributions of MKT, SMB, HML (HMLM or VPD), UMD, CMA, RMW, QMJ, IA, ROE to $Sh^2(f)$. The six base models include AF3 (Asness and Frazzini (2013) 3-factor model), Carhart (1997) 4-factor, Fama and French (2015) 5-factor, FF 5-factor plus momentum factor, and AFP5 (Carhart (1997) 4-factor plus Asness et al. (2019) Quality-minus-Junk factor), and BS6 (Barillas and Shanken (2018) 6-factor model) models. The alternative models are the base models with HML or HMLM replaced with VPD. The VPD factor is defined as in Section 4 and Table 6. All standard errors are calculated using White (1980) t-statistics.

uses pricing errors from a “constrained” cross-sectional regression where the estimates for factor risk premiums are forced to be equal to factor means instead of being freely estimated in the OLS regression. The constrained R-squared evaluates the explanatory power of factor models more accurately because it is based on the correct factor risk premium estimates, while the traditional cross-sectional OLS R-squared overstates the true explanatory power of models that only include traded factors since it relies on implausible estimates of factor risk premiums.

Following Maio (2019), we derive the constrained R-squared from the traditional two-step regression where the risk premiums are forced to be equal to the factor means in the cross-sectional regression of the second step. For illustration purposes, consider the standard Fama and French (1993) 3-factor model.

In the first step, factor betas $\hat{\beta}_i$ are estimated from a time series regression for each asset or portfolio,

$$R_{i,t} - R_{f,t} = \gamma_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \epsilon_{i,t}, \tag{4}$$

where $R_{i,t}$ is the return on asset i at time t ; $R_{f,t}$ is the risk-free rate at time t , MKT, SMB, and HML denote the market risk premium, size factor, and value factor, respectively, in the Fama and French (1993) 3-factor model, whereas $\beta_{i,MKT}$, $\beta_{i,SMB}$, and $\beta_{i,HML}$ denote the corresponding factor loadings for asset i . γ_i and $\epsilon_{i,t}$ denote the intercept and residual, respectively.

In the second step, each factor risk premium λ is estimated by an OLS cross-sectional regression:

$$\overline{R_i - R_f} = \lambda_{MKT} \hat{\beta}_{i,MKT} + \lambda_{SMB} \hat{\beta}_{i,SMB} + \lambda_{HML} \hat{\beta}_{i,HML} + \delta_{i,OLS}, \tag{5}$$

where $\overline{R_i - R_f}$ denotes the time series average excess return for asset i ; λ_{MKT} , λ_{SMB} , and λ_{HML} denote the risk premium for the market, size, and value factors, respectively; and $\delta_{i,OLS}$ denotes the residual. The traditional measure of the goodness-of-fit is the cross-sectional OLS R-squared:

$$R_{OLS}^2 = 1 - \frac{Var(\delta_{i,OLS})}{Var(\overline{R_i - R_f})}, \tag{6}$$

where $Var(\cdot)$ denotes the cross-sectional variance. Since an intercept is not included in the cross-sectional regression, this R-squared measure may take negative values, which implies that adding factor loadings as regressors will generate worse performance than simply adding an intercept in a regression (i.e., the factor model performs worse than a simple model that predicts constant risk premium in the cross-section of average stock returns).

By definition, when factors in a model are excess returns, the model should also price these factors. This implies that the risk premium estimates in cross-sectional regression Eq. (5) correspond to the sample means of the factors rather than being freely estimated in a cross-sectional regression. Thus, the “constrained” cross-sectional regression where the factor risk premium estimates are equal to the factor sample means is:

$$\overline{R_i - R_f} = \overline{MKT} \hat{\beta}_{i,MKT} + \overline{SMB} \hat{\beta}_{i,SMB} + \overline{HML} \hat{\beta}_{i,HML} + \delta_{i,C}, \tag{7}$$

where \overline{MKT} , \overline{SMB} , and \overline{HML} denote the sample means of the *MKT*, *SMB*, and *HML* factors, respectively, and $\delta_{i,C}$ denotes the “correct” residual or pricing error of the regression. The “constrained” cross-sectional R-squared is then defined as:

$$R_C^2 = 1 - \frac{Var(\delta_{i,C})}{Var(\overline{R_i - R_f})}, \tag{8}$$

where R_C^2 will be significantly smaller than the OLS counterpart if the factor risk premiums estimated from the cross-sectional regression significantly differ from sample means of factors. Conversely, R_C^2 will be similar to its OLS counterpart if the factor risk premium estimates are close to the factor means (i.e., the constraint is not binding). Following [Maio \(2019\)](#), we evaluate the statistical significance of the sample R_C^2 and of the spread $R_{C,1}^2 - R_{C,2}^2$ between models 1 and 2 by computing p -values based on 5000 bootstrap simulations, where the residuals from the time series regressions and the risk factor realizations are simulated independently and the contemporaneous cross-sectional correlations between asset returns and between factors are preserved.⁷ The p -values of R-squared are calculated as the fractions of artificial samples in which the pseudo-explanatory ratio is higher than the sample estimate, and those of the spreads are calculated as the fractions of simulated samples in which the pseudo spread is higher (lower) than the sample spread if it is positive (negative).

Finally, we compare several multifactor models by examining their ability, measured by the constrained R-squared, to explain the cross-section of portfolio returns associated with 12 commonly discussed anomalies that could not be explained well by CAPM. The competing models include six base models (AF3, Carhart, FF5, FF5+UMD, AFP5, and BS6), and six alternative models that replace *HML* or *HMLM* with the value-price divergence factor (*VPD*) in each of the base models (AF3_VPD, Carhart_VPD, FF5_VPD, FF5+UMD_VPD, AFP5_VPD, and BS6_VPD). For comparison, we also include the CAPM, HMXZ5, and SY4 models. The test portfolios are 10 deciles of stocks sorted by each of the following 12 firm characteristics: *V/P*, *ME*, *B/M*, *OP*, *Inv*, *NS*, *Ac/B*, *Beta*, *Var*, *RVar*, *Mom*, and *Turnover*. That is, for each of the 12 firm characteristics, stocks are split into 10 deciles according to the ranking of their values at the

end of each month, and then the month-end market cap weighted return of all stocks in that group in the next month is calculated as that group’s return in the next month. The portfolios are rebalanced monthly. To compare the joint explanatory power on firm characteristics, we also include a big portfolio containing all firm characteristic-sorted portfolios, a total of 120 groups of stocks, as testing portfolios. All portfolio sorts use NYSE breakpoints and the sample period is from July 1978 to June 2018.

[Table 11](#) presents the constrained R-squared R_C^2 for regressions of each group of test portfolio returns on various factors. Panel A provides the results for the joint test portfolios, which pool together all firm characteristic portfolios, and Panel B displays the results for single characteristic-sorted test portfolios. We focus on the joint test results as they are more robust due to more observations in the regressions.

The joint test results for R_C^2 in Panel A of [Table 11](#) show that only the Carhart and Carhart_VPD models have positive R_C^2 in the joint test. The R_C^2 for the Carhart_VPD is the highest at 30%, statistically significant at 99% level, and that for the Carhart model is 14%, statistically significant at 95% level. However, the other models have negative R_C^2 ranging from -58% to -19% . The findings suggest that the Carhart_VPD model provides the highest explanatory power for the cross-sectional stock returns as measured by the constrained R-squared R_C^2 .

The results for single portfolio tests listed in Panel B of [Table 11](#) are similar, although weaker, due to fewer observations in each test. First, we find that most models have negative R_C^2 . For the two best models in the joint test, the Carhart model has positive R_C^2 only in tests for *ME*, *B/M*, *Mom*, *Inv*, and *Var* portfolios, where the first three sorting variables correspond to the model’s *SMB*, *HML*, and *Mom* factors while the Carhart_VPD model has positive R_C^2 only in tests for the *V/P*, *ME*, *Mom*, *OP*, *Var*, and *RVar* anomalies, where the first three sorting variables correspond to the model’s *VPD*, *SMB*, and *UMD* factors. Across all single portfolio tests, we find that multi-factor models provide the highest explanatory power for the *V/P* and *B/M* portfolios: 6 out of 14 models have positive R_C^2 in the *V/P* portfolio tests, while only 4 models have positive R_C^2 in the *B/M* portfolio tests. While they provide low explanatory power for the *NS*, *Beta*, and *Turnover* portfolios, all multi-factor models have negative R_C^2 in the single portfolio test associated with each firm characteristic. Not surprisingly, *VPD*-related factor models all have positive R_C^2 in the *V/P* portfolio test as the *VPD* factor is constructed using *V/P* sorts. Furthermore, for each of the remaining portfolio tests, only 1 - 3 models have positive R_C^2 .

We next examine whether replacing *HML* with *VPD* statistically increases models’ explanatory power as measured by R_C^2 . We compare models by computing the pairwise difference of R_C^2 between models before and after *HML* or *HMLM* is replaced by *VPD*. We also compare models that include the *VPD* factor with models that do not contain *HML* factors. The test portfolios remain unchanged and the statistical significance of the spread is measured by the bootstrapped p -value.

[Table 12](#) presents the pairwise R_C^2 differences between six alternative models and six base models in single portfolio tests (Panel B) and the joint test (Panel A). The six base models (AF3, Carhart, FF5, FF5+UMD, AFP5, and BS6) include the *HML* or *HMLM* factor, and the six alternative models are their corresponding models with *HML* or *HMLM* replaced by *VPD*. The results are mixed. Replacing *HML* by *VPD* significantly increases the R_C^2 for the AF3, Carhart, and AFP5 models, but significantly decreases R_C^2 for the FF5 and FF5+UMD models. For the BS6 model, the change has no significant impact on the R_C^2 . Specifically, for AF3, Carhart, and AFP5 models, replacing *HML* by *VPD* significantly increases their R_C^2 by 28%, 16%, and 21% (all in absolute term) in the joint test, respectively, and increases R_C^2 in 6, 4, and 7 tests out of 12 single portfolio tests, respectively. For the FF5 and FF5+UMD models, replacing *HML* by

⁷ See details of the bootstrap simulation in the “Bootstrap simulation” section in [Maio \(2019\)](#)

Table 11
Constrained R-Squared Estimates: 1978–2018.

Model	Panel A:						Panel B: Single Portfolio Test						
	Joint Test	V/P	ME	B/M	OP	Inv	NS	Ac/B	Beta	Var	RVar	Mom	Turnover
CAPM	-0.56	-0.55	0.01	-1.12	-0.32	-1.5	-0.68	-0.27	-7.23	-2.33	-4.95	0.05	-2.2
AF3	-0.58	-0.24	0.08	-0.02	-0.86	-0.41	-0.58	-0.41	-7.2	-2.56	-5.46	-0.98	-1.84
AF3_VPD	-0.3	0.76***	0.01	0.02	-0.3	-0.43	-0.53	-0.73	-2.23	-0.13	-1.57	-2.17	-0.61
Carhart	0.14**	-0.66	0.31***	0.42***	-0.21	0.43***	-0.16	-0.14	-1.5	0.68***	0	0.53***	0.04
Carhart_VPD	0.30***	0.74***	0.17*	-0.38	0.32***	-1.11	-0.41	-0.26	-0.36	0.46**	0.71***	0.53***	0.19
HMXZ5	-0.42	0	0.18	0	-1.19	-1.73	-0.57	0.44***	-4.48	-3.02	-5.78	-0.47	-1.5
FF5	-0.24	-0.16	0	0.29**	-0.19	-1.48	-1.08	-0.74	-0.39	-0.57	-0.95	-0.06	-2.51
FF5_VPD	-0.58	0.60***	-0.07	-0.38	-0.5	-2.39	-1.38	-1.14	-0.72	-1.52	-1.84	-1.53	-4.86
FF5+UMD	-0.22	-0.5	0.14	0.19	-0.14	-1.27	-0.89	-0.42	-1.07	-1.58	-2.15	0.19	-2.07
FF5+UMD_VPD	-0.41	0.64***	0.13	-0.7	-0.39	-2.56	-1.16	-0.59	-3.62	-4.18	-4.55	-0.03	-4.68
AFP5	-0.4	-0.47	0.11	0.27*	-0.99	-0.66	-0.61	-1.15	-2.35	-3.99	-5.43	0.09	-2.22
AFP5_VPD	-0.19	0.63***	-0.01	-2.65	-1.25	-2.7	-0.28	-0.73	-1.57	-2	-1.87	0.31*	-0.42
SY4	-0.26	-0.31	0.38**	0.61***	0.12	-0.71	-0.5	-0.14	-2.19	-3.29	-4.87	-0.8	-3.4
BS6	-0.42	0.03	0.11	-1.16	-0.79	-2.14	-1.11	-0.89	-1.15	-3.29	-4.25	-0.04	-3.46
BS6_VPD	-0.4	0.61***	0.17	-0.78	-0.54	-2.98	-1.27	-0.78	-2.74	-4.22	-4.54	-0.05	-4.53

This table reports the constrained R-squared R_c^2 estimates for 15 factor models using firm characteristics sorted portfolios as test assets. $R_c^2 = 1 - \frac{Var(\delta_{i,C})}{Var(\bar{R}_i - R_f)}$ is estimated from a standard two-step regression: In the first step, for each portfolio i , factor betas $\hat{\beta}_{i,k}$ are estimated from a time series regression: $R_{i,t} - R_{f,t} = \alpha_i + \sum_{k=1}^K \beta_{i,k} f_{k,t} + \epsilon_{i,t}$, where $R_{i,t}$, $R_{f,t}$, $f_{k,t}$ are monthly return of the portfolio i , the risk-free asset, and the factor k . In the second step, a “constrained” cross-sectional regression is ran: $\bar{R}_i - R_f = \sum_{k=1}^K \hat{\beta}_{i,k} \bar{f}_k + \delta_{i,C}$, where $\bar{R}_i - R_f$ represents the time series average excess return of portfolio i and \bar{f}_k is the time series average return of factor k . $\epsilon_{i,t}$ and $\delta_{i,C}$ are the residuals in two regressions. The 15 models include six base models: AF3 (Asness and Frazzini (2013) 3-factor model); Carhart (1997) 4-factor, Fama and French (2015) 5-factor; FF 5-factor plus momentum factor; AFP5 (Carhart (1997) 4-factor plus Asness et al. (2019) Quality-minus-Junk factor), and BS6 (Barillas and Shanken (2018) 6-factor model) models; six alternative models that replace HML or HMLM with the value-price divergence factor (VPD) in each of the base model and are labeled with VPD in their names, basic CAPM, HMXZ5 (Hou et al. (2021) q5-factor model), and SY4 (Stambaugh and Yuan (2016) 4-factor mispricing model). The VPD factor is defined as in Section 4 and Table 6. Panel A presents the results for joint tests where all single test portfolios are pooled together as test assets, and Panel B presents the results for single portfolio tests where the test assets in each test are 10 deciles sorted by each of the V/P, ME, B/M, OP, Inv, NS, Ac/B, Beta, Var, RVar, Mom and Turnover firm characteristics, whose definitions are listed in the appendix. *, **, *** indicate significance at 10%, 5%, 1% significance level, respectively, based on the empirical p -values from 5000 bootstrap simulation (see Maio (2019) for details). The sample period is from June 1978 to June 2018.

Table 12
Constrained R-Squared Difference Estimates: Replacing HML with VPD: 1978–2018.

Alternative Model	Base Model	Panel A: Joint Test	Panel B: Single Portfolio Test	B/M	OP	Inv	NS	Ac/B	Beta	Var	RVar	Mom	Turnover
AF3_VPD	AF3	0.28***	-0.07	0.03	0.56***	-0.02	0.04	-0.32**	4.97***	2.43**	3.89***	-1.18***	1.23***
Carhart_VPD	Carhart	0.16**	-0.13	-0.80***	0.53***	-1.53***	-0.25*	-0.12	1.14***	-0.22	0.71***	-0.01	0.15
FF5_VPD	FF5	-0.34***	-0.08	-0.67***	-0.31***	-0.91***	-0.30***	-0.41***	-0.33**	-0.95***	-0.89***	-1.48***	-2.36***
FF5+UMD_VPD	FF5+UMD	-0.20**	0	-0.89***	-0.25**	-1.28***	-0.27**	-0.16*	-2.55***	-2.61***	-2.40***	-0.22*	-2.61***
AFP5_VPD	AFP5	0.21**	-0.12	-2.93***	-0.26*	-2.04***	0.34**	0.42***	0.78**	1.99***	3.56***	0.22	1.80***
BS6_VPD	BS6	0.02	0.06	0.38**	0.25**	-0.84***	-0.16*	0.11	-1.60***	-0.93***	-0.3	-0.01	-1.07***

This table reports the difference in constrained R-squared estimates $R_{C,alternative}^2 - R_{C,base}^2$ between a base value model and its alternative model that replaces HML or HMLM factor in the base model with the VPD factor. $R_C^2 = 1 - \frac{Var(\hat{\delta}_{i,t})}{Var(R_{i,t} - R_{f,t})}$ is estimated from a standard two-step regression: In the first step, for each portfolio i , factor betas $\hat{\beta}_{i,k}$ are estimated from a time series regression: $R_{i,t} - R_{f,t} = \alpha_i + \sum_{k=1}^K \beta_{i,k} f_{k,t} + \epsilon_{i,t}$, where $R_{i,t}$, $R_{f,t}$, $f_{k,t}$ are monthly return of the portfolio i , the risk-free asset, and the factor k . In the second step, a “constrained” cross-sectional regression is ran: $\bar{R}_i - \bar{R}_f = \sum_{k=1}^K \hat{\beta}_{i,k} \bar{f}_k + \delta_{i,C}$, where $\bar{R}_i - \bar{R}_f$ represents the time series average excess return of portfolio i and \bar{f}_k is the time series average return of factor k . $\epsilon_{i,t}$ and $\delta_{i,C}$ are the residuals in two regressions. The six base models are AF3 (Asness and Frazzini (2013) 3-factor model), Carhart (1997) 4-factor, Fama and French (2015) 5-factor, FF 5-factor plus momentum factor, AFP5 (Carhart (1997) 4-factor plus Asness et al. (2019) Quality-minus-junk factor), and BS6 (Barillas and Shanken (2018) 6-factor) models. The alternative models are labeled with VPD in their names. The VPD factor is defined as in Section 4 and Table 6. Panel A presents the results for joint tests where all single test portfolios are pooled together as test assets, and Panel B presents the results for single portfolio tests where the test assets in each test are 10 deciles sorted by each of the V/P, ME, B/M, OP, Inv, NS, Ac/B, Beta, Var, RVar, Mom and Turnover firm characteristics, whose definitions are listed in the appendix. *, **, *** indicate significance at 10%, 5%, 1% level, respectively, based on the empirical p-values from 5000 bootstrap simulation [see Miao (2019) for details]. The sample period is from June 1978 to June 2018.

VPD significantly decreases their R_C^2 by 34% and 20% (all in absolute term) in the joint test, respectively, and decreases the R_C^2 in 11 and 10 tests out of 12 single portfolio tests, respectively. For BS6 model, the R_C^2 spread is 2% but not statistically different from zero in the joint test.

We compare the six models containing the VPD factor with the HMXZ5 and the SY4 models, which do not contain the value factor (HML), by computing the pairwise R_C^2 spreads. The results in the first 12 rows of Table 13 show that the Carhart_VPD model is the only model that dominates both the HMXZ5 and SY4 models. In the joint test, the R_C^2 of Carhart_VPD is 61% and 53% higher than HMXZ5 and SY4, respectively, and both increases are statistically significant at the 99% level. In 12 single portfolio tests, 8 (6) of the R_C^2 spreads are positively significant for the HMXZ5 (SY4) model. At the other extreme, the FF5_VPD is dominated by both the HMXZ5 and the SY4 models. In the joint test, the R_C^2 spread between the FF5_VPD and HMXZ5 models is -26% and it is -33% between the FF5_VPD and SY4 models. Both are statistically significant at the 99% level. In addition, most of the R_C^2 spreads in the single portfolio tests are negative and statistically significant. The results for the other models are mixed: the joint tests and the single portfolio tests for the AF3_VPD and AFP5_VPD models are not significantly different from the HMXZ5 and SY4 models, while the SY4 model dominates both the FF5+UMD_VPD and BS6 models.

As the Carhart_VPD model dominates both the HMXZ5 and the SY4 models, we next examine whether its base model, the Carhart model, also dominates both models. We compute the corresponding R_C^2 spreads and give the results in the last two rows of Table 13. The results show that the baseline Carhart model also dominates both the HMXZ5 and SY4 models. However, the R_C^2 spreads are lower than those for Carhart_VPD model. In the joint test, the R_C^2 spread between the Carhart and HMXZ5 models and between the Carhart and SY4 models is 45% and 41%, respectively, both statistically significant at the 99% level, and 12% and 16% lower than those for the Carhart_VPD model, respectively. Similar to the Carhart_VPD model, in the 12 single portfolio tests, 9 (7) of the R_C^2 spreads between the Carhart and HMXZ5 (SY4) model are positively significant at the 90% level.

Finally, we run a comprehensive pairwise comparison between the Carhart_VPD model, the best model (see in Tables 11–13), and all the following models: AF3, FF5, FF5+UMD, AFP5, BS6, AF3_VPD, FF5_VPD, FF5+UMD_VPD, AFP5_VPD, and BS6_VPD. Table 14 presents the pairwise R_C^2 spreads. The results show that the Carhart_VPD model is the strongest model. In the joint test, the R_C^2 spreads range from 49% to 88%, all statistically significant at the 99% level. In the single portfolio tests, 76% of the R_C^2 spreads are significantly positive at the 90% level.

In summary, the constrained R-squared test reveals that the Carhart_VPD model entailing MKT, SMB, VPD, and UMD factors explains the cross-sectional stock returns better than the well-known multi-factor models including the AF3, Carhart, FF5, FF5+UMD, HMXZ5, AFP5, SY4, BS6 models, and their corresponding models where HML or HMLM is replaced with VPD. The Carhart_VPD model has the highest R_C^2 and dominates the other models as indicated by the significantly positive R_C^2 spreads.

6. Robustness to alternative specifications of costs of capital

Cost of capital plays an important role in RIM valuation, and consequently in the discussion of the RIM-based value premium and factor pricing with VPD. In this section, we replicate our main tests using different definitions of the cost of capital in calculating the intrinsic value V of a stock (Eq. (2)).

Our findings remain robust when we employ either a constant cost of capital (with values of 11%, 12%, or 13%) or an industry-specific cost of capital estimated in a Bayesian framework using

Table 13
Constrained R-Squared Difference Estimates: VPD models vs. models w/o HML: 1978–2018.

Model 1	Model 2	Panel A: Joint Test	Panel B: Single Portfolio Test											
			V/P	ME	B/M	OP	Inv	NS	Ac/B	Beta	Var	RVar	Mom	Turnover
AF3_VPD	HMXZ5	0.01	1.29***	-0.01	0.06	0.06	1.09***	0.45**	-0.12	-1.64***	2.07***	1.32**	-2.38***	1.74***
AF3_VPD	SY4	0.01	1.11***	-0.37**	-0.39*	-0.37**	0.33	0	-0.75***	0.15	3.41***	3.90***	-1.72***	2.41***
Carhart_VPD	HMXZ5	0.61***	1.26***	0.15	-0.34*	0.68***	0.41*	0.58***	0.35**	0.23	2.67***	3.60***	0.31	2.54***
Carhart_VPD	SY4	0.53***	1.08***	-0.21	-0.67**	0.16	-0.18	0.12	-0.22	1.67**	3.64***	5.54***	1.19***	3.32***
FF5_VPD	HMXZ5	-0.26***	1.12***	-0.09	-0.34*	-0.14	-0.88***	-0.39***	-0.54***	-0.13	0.69**	1.05**	-1.75***	-2.51***
FF5_VPD	SY4	-0.33***	1.00***	-0.44***	-0.44**	-0.77***	-0.84***	-0.73***	-1.14***	0.75*	1.07**	2.05***	-1.05***	-4.24***
FF5+UMD_VPD	HMXZ5	-0.1	1.16***	0.12	-0.66**	-0.03	-1.04***	-0.17	0.02	-3.04***	-1.97***	-1.66***	-0.25	-2.32***
FF5+UMD_VPD	SY4	-0.24**	1.02***	-0.23**	-0.66**	-0.68***	-0.97***	-0.52***	-2.54***	-2.23***	-1.33**	0.46*	-4.15***	
AFP5_VPD	HMXZ5	0.13	1.15***	-0.03	-2.61***	-0.89***	-1.19***	0.71***	-0.12	-0.99**	0.21	1.02**	0.1	1.93***
AFP5_VPD	SY4	0	1.01***	-0.40**	-2.56***	-1.70***	-1.43***	0.29*	-0.75***	0.08	0.72*	2.25***	0.96***	2.47***
BS6_VPD	HMXZ5	-0.08	1.13***	0.15**	-0.74***	-0.19*	-1.46***	-0.28**	-0.17	-2.16***	-2.01***	-1.65***	-0.27*	-2.17***
BS6_VPD	SY4	-0.28**	0.99***	-0.20*	-0.72**	-0.88***	-1.44***	-0.67***	-0.76***	-1.97***	-2.66***	-1.88***	0.35	-4.63***
Carhart	HMXZ5	0.45***	-0.14	0.29**	0.46*	0.15	1.94***	0.83***	0.47**	-0.92***	2.88***	2.88***	0.32*	2.39***
Carhart	SY4	0.41***	-0.24**	-0.06	0.07	-0.34***	1.27***	0.44**	-0.13	1.04**	4.08***	5.17***	1.20***	3.24***

This table reports the spreads of constrained R-squared estimates $R_{c,1}^2 - R_{c,2}^2$ between models including value factor (Model 1) and models do not have value factor (Model 2). $R_C^2 = 1 - \frac{\text{Var}(\hat{\delta}_{i,C})}{\text{Var}(R_i - R_f)}$ is estimated from a standard two-step regression: In the first step, for each portfolio i , factor betas $\hat{\beta}_{i,k}$ are estimated from a time series regression: $R_{i,t} - R_{f,t} = \alpha_i + \sum_{k=1}^K \hat{\beta}_{i,k} f_{k,t} + \epsilon_{i,t}$, where $R_{i,t}$, $R_{f,t}$, $f_{k,t}$ are monthly return of the portfolio i , the risk-free asset, and the factor k . In the second step, a “constrained” cross-sectional regression is ran: $\overline{R_i - R_f} = \sum_{k=1}^K \hat{\beta}_{i,k} \bar{f}_k + \delta_{i,C}$, where $\overline{R_i - R_f}$ represents the time series average excess return of portfolio i and \bar{f}_k is the time series average return of factor k . $\epsilon_{i,t}$ and $\delta_{i,C}$ are the residuals in two regressions. Model 1 includes the Carhart model, and models that replace HML or HMLM with VPD factor in the following six models: AF3 (Asness and Frazzini (2013) 3-factor model), Carhart (1997) 4-factor, Fama and French (2015) 5-factor, FF 5-factor plus momentum factor, AFP5 (Carhart (1997) 4-factor plus Asness et al. (2019) Quality-minus-Junk factor), and BS6 (Barillas and Shanken (2018) 6-factor model) models. Model 2 considers HMXZ5 (Hou et al. (2021) q5-factor model) and SY4 (Stambaugh and Yuan (2016) 4-factor mispricing model) models. The VPD factor is defined as in Section 4 and Table 6. Panel A presents the results for joint tests where all single test portfolios are pooled together as test assets, and Panel B presents the results for single portfolio tests where the test assets in each test are 10 deciles sorted by each of the V/P, ME, B/M, OP, Inv, NS, Ac/B, Beta, Var, RVar, Mom and Turnover firm characteristics, whose definitions are listed in the appendix. *, **, *** indicate significance at 10%, 5%, 1% level, respectively, based on the empirical p-values from 5000 bootstrap simulation (see Maio (2019) for details). The sample period is from June 1978 to June 2018.

Table 14
Constrained R-Squared Difference Estimates: Carhart_VPD vs. Others: 1978–2018.

Model 1	Model 2	Panel A: Joint Test										Panel B: Single Portfolio Test													
		V/P	ME	B/M	OP	Inv	NS	Ac/B	Beta	Var	RVar	Mom	Turnover	V/P	ME	B/M	OP	Inv	NS	Ac/B	Beta	Var	RVar	Mom	Turnover
Carhart_VPD	AF3	0.98**	0.09	-0.37	1.18**	-0.70**	0.17*	0.15	6.84**	3.02**	6.17**	1.51**	2.03**	0.88**	0.09	-0.37	1.18**	-0.70**	0.17*	0.15	6.84**	3.02**	6.17**	1.51**	2.03**
Carhart_VPD	FF5	0.90**	0.17	-0.67**	0.52**	0.37*	0.67**	0.48**	0.03	1.03*	1.66**	0.59**	2.69**	0.54**	0.17	-0.67**	0.52**	0.37*	0.67**	0.48**	0.03	1.03*	1.66**	0.59**	2.69**
Carhart_VPD	FF5+UMD	1.24**	0.04	-0.57**	0.46**	0.17	0.48**	0.17	0.71**	2.03**	2.85**	0.34*	2.26**	0.51**	0.04	-0.57**	0.46**	0.17	0.48**	0.17	0.71**	2.03**	2.85**	0.34*	2.26**
Carhart_VPD	APP5	1.21**	0.06	-0.65**	1.31**	-0.45**	0.2	0.90**	1.99**	4.45	6.14**	0.44*	2.41**	0.70**	0.06	-0.65**	1.31**	-0.45**	0.2	0.90**	1.99**	4.45	6.14**	0.44*	2.41**
Carhart_VPD	BS6	0.71**	0.06	0.78**	1.12**	0.70**	0.79*	3.75**	0.79*	3.75**	4.95**	0.57**	3.65**	0.71**	0.06	0.78**	1.12**	0.70**	0.79*	3.75**	0.79*	3.75**	4.95**	0.57**	3.65**
Carhart_VPD	AF3_VPD	0.60**	0.16**	-0.40**	0.62**	-0.68**	0.12*	0.48**	1.87**	0.59**	2.28**	2.69**	0.80**	0.60**	0.16**	-0.40**	0.62**	-0.68**	0.12*	0.48**	1.87**	0.59**	2.28**	2.69**	0.80**
Carhart_VPD	FF5_VPD	0.88**	0.14	0	0.83**	1.28**	0.97**	0.89**	0.36	1.98**	2.55**	2.06**	5.05**	0.88**	0.14	0	0.83**	1.28**	0.97**	0.89**	0.36	1.98**	2.55**	2.06**	5.05**
Carhart_VPD	FF5+UMD_VPD	0.71**	0.04	0.32*	0.71**	1.45**	0.75**	0.33*	3.26**	4.64**	5.26**	0.56**	4.87**	0.71**	0.04	0.32*	0.71**	1.45**	0.75**	0.33*	3.26**	4.64**	5.26**	0.56**	4.87**
Carhart_VPD	APP5_VPD	0.49**	0.18*	2.27**	1.57**	1.59**	-0.13*	0.47**	1.21**	2.46**	2.58**	0.21	0.61*	0.49**	0.18*	2.27**	1.57**	1.59**	-0.13*	0.47**	1.21**	2.46**	2.58**	0.21	0.61*
Carhart_VPD	BS6_VPD	0.69**	0.13	0.40*	0.87**	1.87**	0.86**	0.53**	2.38**	4.68**	5.25**	0.58**	4.72**	0.69**	0.13	0.40*	0.87**	1.87**	0.86**	0.53**	2.38**	4.68**	5.25**	0.58**	4.72**

This table reports the spreads of constrained R-squared estimates $R^2_{Carhart_VPD} - R^2_{Model 2}$ estimates between Carhart_VPD (Carhart 4-factor model with HML replaced with VPD) and each of the following 10 value models (Model 2): AF3 (Asness and Frazzini (2013) 3-factor model), Fama and French (1997) 4-factor plus momentum factor, APP5 (Carhart (1997) 4-factor plus Asness et al. (2019) Quality-minus-lunk factor), and BS6 (Barillas and Shanken (2018) 6-factor model) models, and their corresponding alternative models that replace HML or HMLM with VPD and are labeled with VPD in their names. The constrained R-squared $R^2_C = 1 - \frac{Var(\hat{\beta}_k)}{Var(R_k - \bar{R}_k)}$ is estimated from a standard two-step regression: In the first step, for each portfolio i , factor betas $\hat{\beta}_{i,k}$ are estimated from a time series regression: $R_{i,t} - R_{f,t} = \alpha_i + \sum_{k=1}^K \hat{\beta}_{i,k} f_{k,t} + \epsilon_{i,t}$, where $R_{i,t}$, $R_{f,t}$, $f_{k,t}$ are monthly return of the portfolio i , the risk-free asset, and the factor k . In the second step, a "constrained" cross-sectional regression is ran: $\bar{R}_i - \bar{R}_j = \sum_{k=1}^K \hat{\beta}_{i,k} \bar{f}_k + \delta_{i,j}$, where $\bar{R}_i - \bar{R}_j$ represents the time series average excess return of portfolio i and \bar{f}_k is the time series average return of factor k . $\epsilon_{i,t}$ and $\delta_{i,j}$ are the residuals in two regressions. The VPD factor is defined as in Section 4 and Table 6. Panel A presents the results for joint tests where all single test portfolios are pooled together as test assets, and Panel B presents the results for single portfolio tests where the test assets in each test are 10 deciles sorted by each of the V/P, ME, B/M, OP, Inv, NS, Ac/B, Beta, Var, RVar, Mom and Turnover firm characteristics, whose definitions are listed in the appendix. *, **, *** indicate significance at 10%, 5%, 1% level, respectively, based on the empirical p-values from 5000 bootstrap simulation (see Maio (2019) for details). The sample period is from June 1978 to June 2018.

the Fama and French (1993) 3-factor model as in Pastor and Staambaugh (1999), where the prior of each industry's factor beta is the cross-sectional average of 48 industry betas.

When this Bayesian approach is used in the calculation of V , we find that portfolios sorted based on the V/P ratio generate significant raw returns and alphas, and that the four-factor model that includes MKT , SMB , VPD , and UMD provide strong explanatory power for the cross-section of asset returns, and all remain robust. Specifically, Table A1 in the appendix reports raw returns and alphas for V/P -sorted portfolios when we use Bayesian industry cost of capital. The results are consistent with Table 3 for all models except for the BS6 model, where the long-short portfolio's alpha is positive but not significant. Table A2 contains results for the Fama-MacBeth regression, which are consistent with Table 5. We find that the V/P ratio is significantly positive in the Fama-MacBeth regression that includes common firm characteristics. Similar to Tables 12-14, Table A3 shows that the constrained R-squared of the Carhart_VPD model is significantly higher than that of the other models.

For comparison with the results reported in Table 3, we also calculate the alphas for portfolios sorted by annual book-to-market ratio and monthly book-to-market ratio and present the results in Table A4 and Table A5, respectively. However, we find that the alphas for the long-short annual B/M portfolios are statistically significant only in the CAPM and AF3 models but in none of the other multi-factor models. The alphas for the long-short monthly B/M portfolios are statistically significant on most factor models, but the magnitudes are much smaller than those of the long-short V/P portfolios.

7. Conclusion

We employ the monthly I/B/E/S consensus analysts' earnings forecasts in a three-period accounting-based residual income model to estimate a monthly intrinsic value V of firm equities. We find that the ratio of the intrinsic valuation to the stock price, V/P , contains additional information relative to B/P , and has strong predictive power for future returns, reviving the value premium that seems to have vanished in recent decades. Long-short portfolios based on V/P sorts generate about 7% annualized returns and cannot be explained by common asset pricing factors. We also show sharp improvements in the explanatory power of cross-sectional asset pricing models when the HML value factor is replaced by a value-price-divergence (VPD) factor constructed from V/P sorts. A four-factor model using VPD , market, momentum, and size factors outperforms well-established benchmarks. The findings remain under alternative specifications such as Bayesian based industry-specific cost of capital.

Our findings can be attributed to the fact that V is constructed using analyst consensus forecasts and hence naturally incorporates the market's expectation of a firm's future investment and profitability, as well as real options and intangible assets. Moreover, regardless of whether the RIM-based value premium is a priced risk or simply mispricing, investors can devise profitable value strategies from it, and empirical researchers should substitute the conventional HML value factor with VPD , especially when working with data from recent years.

Credit author statement

We do not have any statement here.

Data availability

Data will be made available on request.

Table A1
Raw Returns and Alphas for V/P Single Sorted Portfolios: Bayesian Industry Cost of Equity, 1978–2018.

Decile	Raw Return	CAPM	AF3	HMXZ5	FF5	FF5+UMD	AFP5	SY4	BS6
1 (low)	0.82	-0.35***	-0.27***	-0.14	-0.14	-0.18*	-0.19*	-0.26**	0.01
2	0.95	-0.07	-0.02	-0.29***	-0.22***	-0.22***	-0.29***	-0.26***	-0.15*
3	0.96	-0.05	-0.03	-0.17	-0.18*	-0.14	-0.19	-0.14	-0.09
4	0.99	0	0.01	-0.22**	-0.23**	-0.22**	-0.25**	-0.15	-0.22**
5	1.01	0.07	0.04	-0.19	-0.24**	-0.20*	-0.20*	-0.12	-0.17*
6	1.05	0.07	0.05	-0.15	-0.16	-0.14	-0.15	-0.12	-0.14
7	1.13	0.15	0.09	-0.18	-0.22*	-0.14	-0.16	-0.11	-0.15
8	1.15	0.13	0.09	-0.15	-0.16	-0.12	-0.12	-0.09	-0.18
9	0.97	0.08	0.03	-0.17	-0.19*	-0.18*	-0.22*	-0.19	-0.24**
10 (high)	1.23	0.38**	0.26**	0.41**	0.22	0.33**	0.36**	0.43**	0.11
High-Low	0.41*	0.73***	0.54***	0.55**	0.37*	0.51***	0.55***	0.69***	0.1

This table replicates table 2 and 3 using Bayesian industry cost of equity (same as Pastor and Stambaugh (1999)) in the calculation of V in Eq. (2). It presents the raw returns of V/P sorted portfolios and the intercepts (alphas) of the time series regressions of monthly excess returns of each V/P sorted portfolio and the long-short portfolio on different factors during July 1978-June 2018: $R_i^t - R_f^t = \alpha_i + \sum_{k=1}^K \beta_{i,k} f_k^t + \epsilon_i^t$, where R_i^t and R_f^t is the month t return of V/P decile i and the risk-free asset, respectively, and f_k^t is the value of kth factor in month t (monthly return for traded factors) in a factor model. In the regression for a long-short portfolio, the dependent variable is the difference between two portfolio returns. Each portfolio is constructed as follows: at the end of each month, stocks are split into ten deciles according to the ranking of V/P. V/P ratio is defined as the fundamental value V calculated on the month-end using a 3-period Residual Income Model (Eq. (2)) divided by the market cap on the month-end. The industry specific cost of equity is estimated in a Bayesian framework using Fama and French (1993) 3-factor model as in Pastor and Stambaugh (1999). Decile 1 (10) includes the 10% stocks with the lowest (highest) V/P, and a long-short portfolio High-Low that buys stocks in decile 10 and shorts stocks in decile 1 is also constructed at the same time. Each portfolio (decile) is then held for 1 month, and its monthly return is calculated as the value-weighted average of stock returns in it. The table presents the raw percentage returns and regression intercepts (alphas in percentage terms) of the following models: basic CAPM, AF3 (Asness and Frazzini (2013) 3-factor model), HMXZ5 (Hou et al. (2021) q5-factor model), FF5 (Fama and French (2015) 5-factor model), FF5+UMD (Fama and French (2015) 5-factor plus Momentum factor model), AFP5 (Carhart (1997) 4-factor plus Asness et al. (2019) Quality-Minus-Junk factor model), SY4 (Stambaugh and Yuan (2016) 4-factor mispricing model), and BS6 (Barillas and Shanken (2018) 6-factor model). All numbers are in percent. *, **, *** indicate significance at 10%, 5%, 1% level, respectively. All t-statistics are White (1980) t-statistics.

Table A2
Fama-MacBeth Regression: Bayesian Industry Cost of Equity, 1978–2018.

Panel A		Adj. R-squared	Int	V/P	ME	B/M	OP Neg	OP Pos	Inv	
Average	0.07***	1.53***	0.16***	-0.00**	0.14*	0.18**	0.39	-0.37**		
t-statistic	25.15	7.72	2.92	-2.26	1.71	2.27	1.35	-2.05		
Panel B		NS	NS Zero	Ac/B Neg	Ac/B Pos	Beta	Var	RVar	Mom	Turnover
Average	-1.26***	0	-0.7	-0.02	0.16**	-0.86	3.69	0.21***	-0.38**	
t-statistic	-3.03	0.12	-1.57	-0.06	2.04	-0.3	1.44	6.51	-2.32	

The table replicates table 5 using Bayesian industry cost of equity (same as Pastor and Stambaugh (1999)) in the calculation of V in Eq. (2) and shows the time series average and t-statistics of the intercepts and slopes of 480 cross-sectional regression of stock i's month t return on its various firm characteristics at month t-1 during July 1978-June 2018: $R_i^t = \alpha^t + \sum_{k=1}^K \beta_k^t X_{i,k}^{t-1} + \epsilon_i^t$, where R_i^t is monthly return of stock i in month t, $X_{i,k}^{t-1}$ is the kth firm characteristic of stock i in month t-1. α^t and β_k^t are the corresponding regression intercepts and coefficients in month t. The firm characteristic in the regressions include V/P, ME, B/M, M, OP Neg (dummy for negative OP), OP Pos (dummy for positive OP), Inv, NS, NS Zero (dummy for zero NS), Ac/B Neg (dummy for negative Ac/B), Ac/B Pos (dummy for positive Ac/B), Beta, Var, RVar, Mom, and Turnover. V/P is the fundamental value V calculated on the month-end using a 3-period Residual Income Model (Eq. (2)) divided by the market cap on the month-end. The definitions for ME, B/M, M, OP, Inv, NS, Ac/B, Beta, Var, RVar, Mom, and Turnover are listed in the appendix. In particular, OP Neg (Ac/B Neg) is one if OP (Ac/B) is negative and zero otherwise, while OP Pos (Ac/B Pos) is OP (Ac/B) if OP (Ac/B) is positive and zero otherwise. NS Zero is one if NS is zero and zero otherwise; Standard errors are baseline Fama and Macbeth (1973) standard errors, *, **, *** indicate significance at 10%, 5%, 1% level, respectively.

Appendix: Variable Definitions

The data we use are CRSP, Compustat, and I/B/E/S, and we define the firm characteristics used in our portfolio sorts in each month t as follows:

ME is the annual market cap, defined as the share price multiplied by the number of shares outstanding, at the end of the latest June.

ME_M is the monthly market cap, defined as the share price multiplied by the number of shares outstanding, at the end of month t.

B/M is the standard annual book-to-market ratio calculated at the end of the latest June. At the end of June in year k, book value is calculated as total assets (Item 6 AT) for the fiscal year-end in year k - 1, minus total liabilities (Item 181 LT), plus balance sheet deferred taxes (Item 74 TXDB), plus balance sheet investment tax

credit (Item 208 ITCB), minus the liquidating value of preferred stock (Item 10 PSTKL) if available, or redemption value of preferred stock (Item 56 PLTKRV), or carrying value of preferred stock (Item 130 PSTK), adjusted for net stock issuance from the fiscal year-end to the end of December of the year k - 1. Annual reports are assumed to be reported six months after the fiscal year end. Market cap is the share price times the number of shares outstanding at the end of December of year k - 1.

B/M_M is the monthly book-to-market ratio calculated at the end of month. Book value is calculated as total assets (Item 6 AT) for the most recent fiscal year-end, minus total liabilities (Item 181 LT), plus balance sheet deferred taxes (Item 74 TXDB), plus balance sheet investment tax credit (Item 208 ITCB), minus the liquidating value of preferred stock (Item 10 PSTKL) if available, or the redemption value of preferred stock (Item 56 PLTKRV), or the carrying value of preferred stock (Item 130 PSTK), adjusted for net

Table A3
Model Comparison using Constrained R-Squared: Bayesian Industry Cost of Equity, 1978–2018.

Panel A			Panel B			Panel C		
Alternative Model	Base Model	Joint Test	Model 1	Model 2	Joint Test	Model 1	Model 2	Joint Test
AF3_VPD	AF3	0.20***	AF3_VPD	HMXZ5	-0.01	Carhart_VPD	AF3	0.69***
Carhart_VPD	Carhart	0.13**	AF3_VPD	SY4	0.02	Carhart_VPD	FF5	0.47***
FF5_VPD	FF5	-0.17***	Carhart_VPD	HMXZ5	0.61***	Carhart_VPD	FF5+UMD	0.39***
FF5+UMD_VPD	FF5+UMD	-0.24***	Carhart_VPD	SY4	0.60**	Carhart_VPD	AFP5	0.66***
AFP5_VPD	AFP5	-0.08	FF5_VPD	HMXZ5	-0.16**	Carhart_VPD	BS6	0.66***
BS6_VPD	BS6	0.14**	FF5_VPD	SY4	-0.19**	Carhart_VPD	AF3_VPD	0.49***
			FF5+UMD_VPD	HMXZ5	-0.15**	Carhart_VPD	FF5_VPD	0.64***
			FF5+UMD_VPD	SY4	-0.24**	Carhart_VPD	FF5+UMD_VPD	0.63***
			AFP5_VPD	HMXZ5	-0.26**	Carhart_VPD	AFP5_VPD	0.74***
			AFP5_VPD	SY4	-0.35**	Carhart_VPD	BS6_VPD	0.52***
			BS6_VPD	HMXZ5	-0.04			
			BS6_VPD	SY4	-0.16*			
			Carhart	HMXZ5	0.48***			
			Carhart	SY4	0.44***			

This table replicates the results in tables 12–14 using Bayesian industry cost of equity (same as Pastor and Stambaugh (1999)) in the calculation of V in Eq. (2). Panel A reports the difference in constrained R-squared estimates $R_{c,alternative}^2 - R_{c,base}^2$ between a base value model and its alternative model that replaces *HML* or *HMLM* factor in the base model with the *VPD* factor. $R_c^2 = 1 - \frac{Var(\delta_{i,c})}{Var(R_i - R_f)}$ is estimated from a standard two-step regression. In the first step, for each portfolio i , factor betas $\hat{\beta}_{i,k}$ are estimated from a time series regression: $R_{i,t} - R_{f,t} = \alpha_i + \sum_{k=1}^K \hat{\beta}_{i,k} f_{k,t} + \epsilon_{i,t}$, where $R_{i,t}$, $R_{f,t}$, $f_{k,t}$ are monthly return of the portfolio i , the risk-free asset, and the factor k . In the second step, a “constrained” cross-sectional regression is ran: $\bar{R}_i - \bar{R}_f = \sum_{k=1}^K \hat{\beta}_{i,k} \bar{f}_k + \delta_{i,c}$, where $\bar{R}_i - \bar{R}_f$ represents the time series average excess return of portfolio i and \bar{f}_k is the time series average return of factor k . $\epsilon_{i,t}$ and $\delta_{i,c}$ are the residuals in two regressions. The six base models are AF3 (Asness and Frazzini (2013) 3-factor model), Carhart (1997) 4-factor, Fama and French (2015) 5-factor, FF 5-factor plus momentum factor, AFP5 (Carhart (1997) 4-factor plus Asness et al. (2019) Quality-minus-Junk factor), and BS6 (Barillas and Shanken (2018) 6-factor) models. The alternative models are labeled with *VPD* in their names. Panel B reports the spreads of constrained R-squared estimates $R_{c,1}^2 - R_{c,2}^2$ between models including value factor (Model 1) and models that do not have value factor (Model 2). Model 1 includes the Carhart model and all the alternative models in Panel A. Model 2 considers HMXZ5 (Hou et al. (2021) q5-factor model) and SY4 (Stambaugh and Yuan (2016) 4-factor mispricing model) models. Panel C reports the spreads $R_{c,Carhart_VPD}^2 - R_{c,2}^2$ estimates between Carhart_VPD (Carhart 4-factor model where *HML* is replaced with *VPD*) and each of the following 10 value models (Model 2): AF3, Fama and French (2015) 5-factor, FF 5-factor plus momentum factor, AFP5, and BS6 models, and their corresponding alternative models that replace *HML* or *HMLM* with *VPD* and are labeled with *VPD* in their names. The *VPD* factor is defined as in Section 4 or Table 6. The “Joint Test” column presents the results for joint tests where the test assets are a pool of 12 groups of 10 deciles sorted by *V/P*, *ME*, *B/M*, *OP*, *Inv*, *NS*, *Ac/B*, *Beta*, *Var*, *RVar*, *Mom* and *Turnover* firm characteristic variables (see appendix for definitions), a total of 120 portfolios. *, **, *** indicate significance at 10%, 5%, 1% level, respectively, based on the empirical p-values from 5000 bootstrap simulation (see Maio (2019) for details). The sample period is from June 1978 to June 2018.

Table A4
Alphas for Annual Book-to-Market Single Sorted Portfolios, 1978–2018.

Decile	CAPM	AF3	HMXZ5	FF5	FF5+UMD	AFP5	SY4	BS6
1 (low)	-0.02	0.06	0.1	0.11*	0.13**	0.04	0.12	0.28***
2	0.12*	0.15**	0.02	0.09	0.11	0.05	0.06	0.18**
3	0.15*	0.16**	-0.1	0	-0.01	-0.08	-0.07	0.02
4	0.06	0.05	-0.14	-0.17*	-0.17*	-0.14	-0.11	-0.20**
5	0.19*	0.16*	-0.01	-0.01	0	0.09	0.04	-0.04
6	0.08	0.04	0.04	-0.07	-0.06	0.04	-0.01	-0.09
7	0.29***	0.24**	0.04	0.01	-0.01	0.04	-0.02	-0.04
8	0.23**	0.17*	0.12	0.01	0.02	0.08	0.09	-0.06
9	0.28**	0.19*	0.01	-0.03	-0.03	0.05	-0.02	-0.11
10 (high)	0.47***	0.37***	0.18	0.08	0.08	0.14	0.06	0.05
High-Low	0.49**	0.31*	0.08	-0.03	-0.05	0.11	-0.06	-0.23

This table presents the intercepts (alphas in percentage terms) of the time series regressions of monthly excess returns of each standard Book-to-Market ratio sorted portfolio and the long-short portfolio on different factors during July 1978–June 2018: $R_i^t - R_f^t = \alpha_i + \sum_{k=1}^K \beta_{i,k} f_k^t + \epsilon_i^t$, where R_i^t and R_f^t is the month t return of Book-to-Market decile i and the risk-free asset, respectively, and f_k^t is the value of k th factor in month t (monthly return for traded factors) in a factor model. In the regression for a long-short portfolio, the dependent variable is the difference between two portfolio returns. Each portfolio is constructed as follows: at the end of each month, stocks are split into ten deciles according to the ranking of Book-to-Market (*B/M*) ratio, which is the standard annual book-to-market ratio calculated at the end of each June as the book value from previous year divided by the market cap at the end of previous year. Decile 1 (10) includes the 10% stocks with the lowest (highest) *B/M*, and a long-short portfolio High-Low that buys stocks in decile 10 and shorts stocks in decile 1 is also constructed at the same time. Each portfolio (decile) is then held for 1 month, and its monthly return is calculated as the value-weighted average of stock returns. The table presents the regression intercepts (alphas in percentage terms) of the following models: basic CAPM, AF3 (Asness and Frazzini (2013) 3-factor model), HMXZ5 (Hou et al. (2021) q5-factor model), FF5 (Fama and French (2015) 5-factor model), FF5+UMD (Fama and French (2015) 5-factor plus Momentum factor model), AFP5 (Carhart (1997) 4-factor plus Asness et al. (2019) Quality-Minus-Junk factor model), SY4 (Stambaugh and Yuan (2016) 4-factor mispricing model), and BS6 (Barillas and Shanken (2018) 6-factor model). *, **, *** indicate significance at 10%, 5%, 1% level, respectively. All t-statistics are White (1980) t-statistics.

Table A5
Alphas for Monthly Book-to-Market Single Sorted Portfolios, 1978–2018.

Decile	CAPM	AF3	HMXZ5	FF5	FF5+UMD	AFP5	SY4	BS6
1 (low)	0.03	0.14**	-0.02	0.14**	0.08	0.01	0.04	0.20***
2	0.03	0.07	-0.12	-0.05	-0.03	-0.09	-0.03	0.07
3	0.08	0.08	-0.12	-0.09	-0.06	-0.1	-0.03	-0.07
4	0.11	0.08	-0.04	-0.1	-0.05	-0.01	0.04	-0.05
5	0.1	0.07	-0.06	-0.12	-0.08	-0.03	-0.03	-0.12
6	0.16	0.11	0.04	-0.09	-0.03	0.01	0.01	-0.05
7	0.23**	0.16	0.08	-0.07	-0.02	0.04	0.07	-0.05
8	0.19*	0.09	0.20*	0.01	0.17*	0.17*	0.20*	0.02
9	0.40***	0.27***	0.45***	0.18	0.32***	0.36***	0.45***	0.19*
10 (high)	0.42**	0.22*	0.63***	0.19	0.44***	0.54***	0.53***	0.35***
High-Low	0.39*	0.09	0.65**	0.05	0.37**	0.53***	0.49**	0.15

This table presents the intercepts (alphas in percentage terms) of the time series regressions of monthly excess returns of each monthly Book-to-Market ratio sorted portfolio and the long-short portfolio on different factors during July 1978–June 2018: $R_i^t - R_f^t = \alpha_i + \sum_{k=1}^K \beta_{i,k} f_k^t + \epsilon_i^t$, where R_i^t and R_f^t is the month t return of monthly Book-to-Market decile i and the risk-free asset, respectively, and f_k^t is the value of k th factor in month t (monthly return for traded factors) in a factor model. In the regression for a long-short portfolio, the dependent variable is the difference between two portfolio returns. Each portfolio is constructed as follows: at the end of each month, stocks are split into ten deciles according to the ranking of B/M_M ratio, which is the monthly book-to-market ratio defined as the most recent reported annual book value divided by the market cap at the end of each month. Decile 1 (10) includes the 10% stocks with the lowest (highest) B/M_M , and a long-short portfolio High-Low that buys stocks in decile 10 and shorts stocks in decile 1 is also constructed at the same time. Each portfolio (decile) is then held for 1 month, and its monthly return is calculated as the value-weighted average of stock returns. The table presents the regression intercepts (alphas in percentage terms) of the following models: basic CAPM, AF3 (Asness and Frazzini (2013) 3-factor model), HMXZ5 (Hou et al. (2021) q5-factor model), FF5 (Fama and French (2015) 5-factor model), FF5+UMD (Fama and French (2015) 5-factor plus Momentum factor model), AFP5 (Carhart (1997) 4-factor plus Asness et al. (2019) Quality-Minus-Junk factor model), SY4 (Stambaugh and Yuan (2016) 4-factor mispricing model), and BS6 (Barillas and Shanken (2018) 6-factor model). *, **, *** indicate significance at 10%, 5%, 1% significance level, respectively. All t-statistics are White (1980) t-statistics.

stock issuance from the most recent fiscal year-end to the end of month t . Annual reports are assumed to be reported six months after the fiscal year end. Market cap is the share price times the number of shares outstanding at the end of month t .

OP is the operating profitability calculated at the end of the latest June as revenues (Item REVT) minus cost of goods sold (Item 41 COGS), minus selling, general, and administrative expenses (Item 132 XSGA), minus interest and related expense (Item 15 XINT) all scaled by book equity. Annual reports are assumed to be reported with a six-month lag.

Inv is the investment factor calculated at the end of the latest June as the relative change in total assets (Item 6 AT) from two years ago to last year.

NS is the net stock issuance factor calculated at the end of the latest June as the total growth in the market cap from the June of previous year to the June of current year divided by the compounded daily without dividend stock returns (CRSP Item RETXD) over the same period, minus 1. NS is zero if CRSP's shares outstanding do not change over this 12-month period.

Ac/B is the standardized accruals calculated at the end of the latest June. At the end of June in year k , it is defined as accruals standardized by book value per split-adjusted share at year $k - 1$, where the accruals are defined as the change in non-cash working capital from $k - 2$ to $k - 1$ at the end of June and non-cash working capital is current assets (Item 4 ACT) minus cash and short-term investments (Item 1 CHE), minus current liabilities (Item 5 LCT), plus debt (Item 34 DLC).

$Beta$ is the beta factor β calculated at the end of the latest June as the sum of the slopes from the regression of monthly returns on the current and first lag of monthly market returns trailing 60 months (24 minimum).

Var is the variance of daily returns estimated monthly using 60 days (20 minimum) of lagged returns.

$RVar$ is the variance of daily residual returns estimated from the Fama and French (1993) 3-factor model using 60 days (20 minimum) of lagged returns. The daily residual return is the stock daily excess return minus sum of the products of factor loadings in the previous day and factor returns on the current day, where the factor loadings are estimated by a rolling 30-day (17 minimum) re-

gression of stock excess returns on Fama-French (1993) three factors (Ang et al., 2006).

Mom is the compounded return during the previous 12-month period with the most recent month skipped.

$Turnover$ is the average daily turnovers during the previous 12-month period.

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