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Persuasion in relationship finance☆

Ehsan Azarmsa[a], Lin William Cong[b,*]

[a] University of Chicago, Economics Department and the Booth School of Business, 5807 S Woodlawn Ave, Chicago, IL 60637, United States
[b] Cornell University Johnson Graduate School of Management, Sage Hall, 106 East Ave, Ithaca, NY 14853, United States

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ABSTRACT

After initial investments, relationship financiers routinely observe interim information about projects before continuing financing them. Meanwhile, entrepreneurs produce information endogenously and issue securities to incumbent insider and competitive outsider investors. In such persuasion games with differentially informed receivers and contingent transfers, entrepreneurs' endogenous experimentation reduces insiders' information monopoly but impedes relationship formation through an "information production hold-up." Insiders' information production and interim competition mitigate this hold-up and jointly explain empirical links between competition and relationship lending. Optimal contracts restore first-best outcomes using convertible securities for insiders and residuals for outsiders. Our findings are robust under various extensions and alternative specifications.

1. Introduction

What benefits do entrepreneurs get by raising capital from intermediaries such as banks or venture funds instead of issuing securities in public markets? A large literature on relationship finance reveals that intermediaries mitigate informational asymmetry and moral hazard by providing initial funding and forming relationships with entrepreneurs to become "insiders" (e.g., Ramakrishnan and Thakor, 1984; Fama, 1985; Diamond, 1991). Researchers recognize how insider financiers' information monopoly may hold up entrepreneurs (e.g., Sharpe,

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* Corresponding author.

E-mail addresses: azarmsa@chicagobooth.edu (E. Azarmsa), will.cong@cornell.edu (L.W. Cong).

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1990; Rajan, 1992) but largely ignore the endogenous nature of information production and design. Yet, in reality, entrepreneurs' actions and experimentation not only shape project cash flows but also alter the informational environment.[1]

Several questions naturally arise. Does the endogenous production of information matter for relationship formation and sequential fund-raising? Can it help rationalize puzzling empirical observations such as the non-monotone relationship between bank orientation and competition (e.g., Degryse and Ongena, 2007) that earlier models cannot explain? What are the implications for designing securities for venture investors at various stages?

To address these questions, we model interim experimentation in relationship financing, contracting, and security design as a Bayesian persuasion game with contingent transfers and differentially informed receivers. Specifically, in our baseline model we consider a capital-constrained (male) entrepreneur with a project requiring two rounds of financing. The first round requires a fixed investment that enables the entrepreneur to "experiment"—broadly interpreted as conducting early stage activities such as hiring key personnel, acquiring initial users, and developing product prototypes—to produce interim information to persuade investors for continued financing. The key friction rests in that the entrepreneur lacks ex-ante commitment to specific experiments and thus to the interim information production. However, by monitoring and having access to the entrepreneur's team through the first-round investment, a relationship financier (henceforth referred to as a female insider or "insider investor") observes and can verify interim signals from the experiment. Because the signals are informative of the eventual profitability, the insider has an advantage relative to arm's-length investors (henceforth referred to as "outsiders"), consistent with assumptions in the literature and observations in practice concerning relationship finance (e.g., Rajan, 1992).

After forming the financing relationship and conducting interim experiments, the entrepreneur raises capital in a second round by issuing securities to the insider and potentially outsiders. The insider enjoys an informational advantage relative to outsiders, who learn from the insider's decision on continuation or termination of the project. We capture the entrepreneur's and investors' (sender's and receivers') divergent interim objectives by modeling the entrepreneur's private benefit of continuation (which is difficult to verify or contract upon), limited liability, and endogenous security choice.

Our key finding is that the entrepreneur's endogenous information production reduces a relationship financier's rent from her interim informational advantage, thus inefficiently holding up her initial investment in the relationship, a phenomenon we call information production hold-up (IPH). We show that the entrepreneur follows a threshold strategy for experimentation to produce information such that the insider is indifferent between termination and continuation. This reduces the insider's information monopoly, rendering the insider incapable of recovering the initial investment in forming the relationship. Good projects thus may fail to get initial financing. We are the first to make these observations, which are in stark contrast to theories on bank monitoring and hold-ups of entrepreneurial effort that take informational environments as exogenous.

Endogenous information productions and their associated hold-ups have two immediate implications. First, IPH challenges the conventional understanding of the roles of investors' information production/acquisition and interim competition in relationship finance. In industries requiring less entrepreneur-specific knowledge, "sophisticated" insiders who use their own information technology to evaluate projects' prospects can extract a positive interim rent and partially restore the feasibility of the initial relationship formation. Relationship financing also becomes viable with moderate interim competition because selling to competitive outsiders encourages more efficient information production by the entrepreneur. Investors' interim competition (reflected through the insider's interim bargaining power) and sophistication (captured by the informativeness of her independent signal) jointly impact the dependence of relationship financing on competition, which is nonmonotone in general. In particular, for intermediate levels of investor sophistication, the ease of relationship formation can therefore depend on interim competition with a U-shaped pattern, consistent with the empirical regularities (Elsas, 2005; Degryse and Ongena, 2007) that other models cannot explain.

The second implication of endogenous information production and IPH lies in how it affects contracting and security design for sequential investors. Even with general investor sophistication and interim competition, entrepreneurs' optimal contracts fully restore efficient information production: the entrepreneur optimally promises the early insider investor convertible securities at a prespecified price and quantity and, upon the insider's continued financing, issues residual securities to the outsiders.[2]

Intuitively, the entrepreneur gets all the ex-ante surplus when facing competitive investors. Upon forming a financing relationship, he wants to, but cannot, commit to efficient interim information production, resulting in an

[1] Pharmaceutical firms can affect the Food and Drug Administration's and investors' decisions by providing additional information and tests (e.g., "Guidance for industry and FDA staff. Postmarket surveillance under Section 522 of the Federal Food, Drug and Cosmetic Act" May 16, 2016). Software start-ups decide on different markets for beta launches because the media attention generated is different and thus information communicated to potential users ("Why is Canada such a good testing ground for game releases?" Forbes Tech. November 27, 2012); entrepreneurs choose the specific prototype or trial market to work on, which produces disparate forms of information; career concerns and managerial choices of projects exhibit similar features.

[2] Our findings do not rely on the entrepreneur's ability to commit to a disclosure policy or designing information to be arbitrarily informative. Instead, we require the entrepreneur can design simple experiments such as one producing binary, threshold signals, and that within a financing relationship an insider verifies interim experiment outcomes better than outsiders.

intertemporal wedge between the ex-ante entrepreneur (who shares the objective of a social planner) and the interim entrepreneur. The mix of inside and outside finance at the interim date then affects not only how surplus is shared (as in Rajan, 1992) but also how much information is produced. The ex-post expropriation (hold-up) generates suboptimal investment decisions by distorting entrepreneurial incentives to produce information. The first-best security design for the entrepreneur should therefore align both his interim information production incentive and the insider's continuation incentive with the social planner's.

This general contractual problem involves an infinitely dimensional nested optimization, prompting us to take a novel constructive proof approach: we first propose a set of contracts restoring social efficiency and then show that they are the only optimal contracts for the entrepreneurs. Outsiders are competitive and pay the entrepreneur fair prices, effectively rendering the entrepreneur the residual claimant of the project's social surplus regardless of the information revealed during the interim. Therefore, giving the insider debt-like securities in all bad states of the world (when the entrepreneur tends to overcontinue) fully exposes the entrepreneur to the cost of inefficient continuation. Meanwhile, relationship financing is feasible as long as the contract yields the insider enough interim rent to recover her initial investment, leaving the security design only partially determinate in good states of the world.

The optimal designs derived are broadly consistent with real life observations in entrepreneurial finance: a variety of convertible securities for insiders and residual securities for arm's-length outsiders. For example, venture capitalists indeed routinely take convertible debts, while arm's-length outsiders receive equities at subsequent public offerings.

Finally, we discuss the robustness of our findings by allowing a restricted experimentation space, partial observability and commitment to disclosure, general investor sophistication, scalable investments, and investors' security design, among others. IPH manifests itself under various security forms, and the economic mechanism applies even beyond relationship lending and staged venture financing. We help underscore and formalize this practical issue and then develop potential contractual solutions. From a theory perspective, our study also provides insights on Bayesian persuasion games with contingent transfers and sequential receivers facing discriminatory disclosures while deepening our understanding of contracting under endogenous information production.

Our theory foremost contributes to the large literature on relationship finance. Theoretical studies on relationship banking focus on interim information and control (e.g., Diamond, 1984; 1991; Fama, 1985). While relationship finance can improve financing efficiency (e.g., Petersen and Rajan, 1994), it naturally induces an information monopoly (Berger and Udell, 1995; Petersen and Rajan, 2002), potentially holding up the entrepreneur's effort in relationship lending (e.g., Rajan, 1992; Santos and Winton, 2008; Schenone, 2010) and venture capital (e.g., Fluck et al., 2006; Ewens et al., 2016). We inform the debate by endogenizing the informational environment and analyzing the IPH problem.[3]

Empirically, the effect of competition on bank orientation has received much attention. Elsas (2005) and Degryse and Ongena (2007) show a puzzling U-shaped effect of market concentration on relationship lending. Extant theories either predict opposing monotone patterns (e.g., Petersen and Rajan, 1995; Boot and Thakor, 2000; Dinc, 2000; Dell'Ariccia and Marquez, 2004) or suggest a hump-shaped pattern (e.g., Yafeh and Yosha, 2001; Anand and Galetovic, 2006). Our theory offers an information-based explanation for the empirical regularities.

We also elucidate the role of intermediaries and security design in entrepreneurial finance.[4] We add to earlier studies on optimal securities in relationship finance or venture capital (e.g., Kaplan and Strömberg, 2004; Hellmann, 2006) by endogenizing general information production and incorporating flexible designs in sequential security issuance and experimentation. The optimality of convertible securities in our paper does not rely on ex-ante informational asymmetry between the issuer and investors (e.g., Stein, 1992; Brennan and Schwartz, 1988), hidden manipulations of signals under exogenously given information structures (Cornelli and Yosha, 2003), or investors' information acquisition (Yang and Zeng, 2018). We are the first to show that first issuing convertible securities to insiders and then equities to outsiders is optimal and robust to investor sophistication and interim competition. We also underscore that the presence of competitive outsiders crucially affects optimal contract designs.

From a theory perspective, our paper contributes to the field of information design, especially Bayesian persuasion (e.g., Kamenica and Gentzkow, 2011; Dworczak and Martini, 2019; Ely, 2017; Guo and Shmaya, 2019). We take a linear programming approach similar to Bergemann and Morris (2016) but allow infinite payoff-relevant states and different types of informed receivers. We do not require that the sender's utility from a message completely depend on the expected state (Kolotilin, 2018) or on the payoff over the receiver's actions to depend on the state linearly (Gentzkow and Kamenica, 2016). More importantly, we incorporate into our model a security design that endogenizes the dependence of the sender and receivers' payoffs on the state, allowing interactions of multiple asymmetrically informed receivers.

By doing so, our paper advances the emerging applications of information design in finance. Despite having been adopted to address issues in banking regulation,

[3] In this regard, our paper broadly relates to incomplete contracting and hold-up problems (e.g., Hart and Moore, 1988; Aghion et al., 1994) and whether long-term contracts can mitigate investment inefficiencies (Von Thadden, 1995; Nöldeke and Schmidt, 1998). We differ primarily in endogenizing interim information production and deriving the optimal design without requiring contractibility of the entrepreneurs' bias of continuation.

[4] Our paper broadly relates to seminal studies examining how agents' actions alter the distribution of cash flows (e.g., Holmstrom, 1979; Innes, 1990). The agent in our setting shapes the informational environment, and securities are issued to heterogeneous agents. While first-best outcomes are typically unattainable in conventional settings, our contractual solution restores efficient information production and investment.

online advertising, entertainment, etc., most Bayesian persuasion models do not allow contingent transfers, which are prevalent in security design and contracting. Relationship finance provides a natural setting for endogenous information design and contingent transfers.[5] Most closely related is Szydlowski (2020), a pioneering study applying Bayesian persuasion to corporate finance, which obtains an irrelevance result of security choice when the entrepreneur jointly designs disclosure and security. Other early applications concern topics on government intervention (Cong et al., 2020), markets for financial advice with heterogeneous agents (Chang and Szydlowski, 2020), and stress tests (Bouvard et al., 2015; Goldstein and Leitner, 2018; Orlov et al., 2019; Inostroza and Pavan, 2018).[6] Our paper instead constitutes the first examination of relationship finance and contracting under endogenous information production.

2. A model of relationship finance and information

2.1. Financing of projects

We consider a three-period economy with time index $t = 0, 1, 2$. There is no time discounting. A risk-neutral entrepreneur has a project that requires a fixed investment $I \in (0, 1)$ at $t = 1$ and produces an uncertain cash flow $X \in [0, 1]$ at $t = 2$ with a prior distribution denoted by a continuous and atomless pdf $f(X)$. The entrepreneur can raise I by issuing securities at $t = 1$ to competitive, risk-neutral investors to finance the project.

In addition, an investor may invest K in the project at $t = 0$ to become a "relationship financier" (the insider). The initial investment enables the entrepreneur to experiment and generate interim information about the distribution of the cash flow. One may also view $K + I$ as the total investment needed but raised in stages whereby early experimentation generates interim information (Kerr et al., 2014). For simplicity, we normalize the cash flow from the seed investment to zero, which is innocuous (see, e.g., Online Appendix OA1).

2.2. Experimentation and information production

The interim experimentation essentially allows the entrepreneur (the sender) to choose messages from a compact metric space $\mathcal{Z}$ and mapping $\pi : [0, 1] \to \Delta(\mathcal{Z})$, where $\Delta(\mathcal{Z})$ is the set of Borel probabilities on the message space. For notational simplicity and without loss of generality, we assume $\mathcal{Z}$ is finite, which implies that the experiment simply involves a mapping described by the conditional probabilities $\pi(z|X)$.[7]

Following the relationship finance literature, we assume that even though $(\mathcal{Z}, \pi)$ is common knowledge, with probability $\mu \in [0, 1]$ only the insider observes the outcome of the experiment, and with probability $1 - \mu$ all investors publicly observe the outcome. μ succinctly captures the extent of the insider's informational advantage through close monitoring and repeated interactions between the entrepreneur and insiders (Megginson and Weiss, 1991); we can interpret $1 - \mu$ as a reduced-form measure of the "interim competition" between the insider and the outsider commonly modeled in the relationship lending literature (Petersen and Rajan, 1995). $\mu = 0$ corresponds to perfect interim competition, and $\mu = 1$ corresponds to information monopoly by the insider.

In our setting, it is not necessary for the entrepreneur to communicate or use a sophisticated disclosure policy. We only need the insider to observe or verify an experiment's outcomes (e.g., reception of a prototype, a trial run, a beta launch, etc.) more easily than the outsiders do. The extent of this advantage is captured by μ. For example, the experiment could be the building and testing of a prototype, over which the entrepreneur has full control. All we require is that an insider understands the prototype test and observes the outcome better.

2.3. Misalignment of incentives

As the project develops after the initial funding, the entrepreneur enjoys a private benefit from continuing the project. Specifically, he receives $\varepsilon \in (0, \bar{\varepsilon})$ if the project is financed at $t = 1$, where $\bar{\varepsilon}$ and K satisfy $\mathbb{E}[(X - I)\mathbb{I}_{\{X \geq I - \bar{\varepsilon}\}}] \geq K$ so that the project is always financed through relationship financing (positive net present value ex ante to the financier) under efficient interim information production. ε has an atomless distribution described by the pdf $g(\cdot)$ ex ante ($t = 0$), and its realization at the start of $t = 1$ is common knowledge.

As we show below, the entrepreneur's limited liability drives most of our results, and therefore we could have alternatively allowed negative values of X that the entrepreneur does not need to bear (e.g., Rajan, 1992). That said, using a private benefit ε to capture the misalignment of incentives is standard in the literature, and it constitutes a realistic source of agency conflict and eases our subsequent exposition of optimal security designs.

To best illustrate our economic mechanism and match reality for early business start-ups, we assume that $\mathbb{E}[X - I + \bar{\varepsilon}] < 0$, which implies the absence of direct financing by arm's-length investors ex ante. This innocuous assumption allows us to focus on the case in which relationship financing and informational considerations are indispensable.

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[5] The commitment assumption has been a major challenge in the field, but here the dynamic financing relationship naturally allows the insider to observe interim experiment and signals. In fact, it is the lack of commitment to information design during entrepreneurs and investors' initial interactions that calls for security design to mitigate inefficient information production.

[6] In a related literature, researchers examine reduced-form parametrized disclosure rules and typically rely on informational asymmetry and signaling or cheap talk. Trigilia (2019), for example, studies company transparency and capital structure in the presence of heterogeneous investors.

[7] As we show in Appendix A1 and Online Appendix OA1, all our results go through had we assumed $\mathcal{Z}$ to be countable or of a continuum or a more restrictive space of information design.

2.4. Contracting environments

In addition to the informational advantage, an insider investor can potentially contract with the entrepreneur upon forming a financing relationship at $t = 0$. We consider two main contracting environments.

First, to highlight the IPH, we assume in Section 3 an exogenously given security $s(X)$, limited liabilities of the entrepreneur and insider investor ($s(X) \in [0, X]$), and double monotonicity for the security (both $s(X)$ and $X - s(X)$ are weakly increasing in X).[8] After z is realized, the insider makes a take-it-or-leave-it offer (TIOLI) to purchase a λ fraction of the security $s(\cdot)$, i.e., $s_I(X) = \lambda s(X)$, at a total price p^I.[9] The entrepreneur decides whether to accept the offer, and then if he still needs financing, he sells the remaining securities, $s_O(X) = (1-\lambda)s(X)$, to outsiders who offer a competitive total price p^O. The investment takes place if and only if I is successfully raised; otherwise the pledged capital is returned to investors.[10]

Second, in Section 4 we allow the entrepreneur to contract at $t = 0$ on both λ and the design of securities to be offered at $t = 1$ to the insider financier and arm's-length investors. This is equivalent to allowing any form of contracts over X, including one that promises the insider $s_0(X)$ at $t = 0$.[11]

We do not allow contracting on the experiment at $t = 0$ for two reasons. First, the security design literature typically makes security payoffs only contingent on cash flows and not on experimentation. Second, in practice a start-up constantly evolves, and it is almost impossible to contract at a seed round or angel round what exact actions the entrepreneurs should take when the founding team is not yet complete, not to mention that investors do not have the same technical expertise or control as the entrepreneurs at such an early stage (e.g., Gompers et al., 2020) and interim signals are often not well-defined and too costly to verify to be useful for security contracts (Kaplan and Strömberg, 2003).

[8] See, e.g., Nachman and Noe (1994), DeMarzo and Duffie (1999), DeMarzo et al. (2005), and Cong (2017). If such monotonicity is violated, either the entrepreneur or the investor can be better off destroying some surplus for some state X, as Hart and Moore (1995) point out.

[9] The TIOLI assumption, albeit natural, is not crucial, and one can alternatively assume that the insider can renegotiate during the interim. The insider proposing a different security can be viewed as a form of renegotiation. Our main results are robust to the possibility of renegotiation after the signal realization because in that case a dominant strategy for the insider is to ask for the whole output X for price p^I. This offer would be equivalent to the specific case of the original security being $s(X) = X$. We discuss this further in Online Appendix OA2.

[10] This scheme is often referred to as "all-or-nothing." Regarding its wide applications and impact on project implementation and information aggregation, see Cong and Xiao (2018).

[11] The insider's getting a total of $s_0(X) + s_I(X)$ upon continuation and nothing otherwise is equivalent to getting some other $s_I(X)$ alone. To highlight the importance of security design, in Online Appendix OA2 we consider a third contracting environment wherein the entrepreneur can contract with the insider at time $t = 0$ only on the fraction λ of security issuance at $t = 1$ that the insider can purchase before it is offered to arm's-length outsiders.

2.5. Interim payoffs and relationship formation

In general, the players' interim payoffs after the formation of a financing relationship are

$$p^O = \mathbb{E}[s_O(X)|\mathcal{F}^O], \tag{1}$$

$$u^E(X; p^I, p^O, \varepsilon) = \left(\varepsilon + X - s_I(X) - s_O(X) + p^I + p^O - I\right)\mathbb{I}_{\{p^I+p^O \geq I\}}, \tag{2}$$

$$u^I(X; p^I, p^O) = \left(s_I(X) - p^I\right)\mathbb{I}_{\{p^I+p^O \geq I\}}, \tag{3}$$

where $\mathcal{F}^O$ denotes the outsiders' information set after having observed the project's continuation or termination (not the insider's offer per se, as the entrepreneur may reject that).[12]

If the project is not financed, all players receive outside options, which are normalized to zero. Intuitively, relationship financing is feasible only if the insider can recover in expectation at least the initial investment K; i.e.,

$$\mathbb{E}[u^I(X; p^I, p^O)] \geq K. \tag{4}$$

Finally, we assume the project would always be funded through relationship financing (positive net present value ex ante to the financier) if the interim information production is socially efficient. This holds under endogenous security design because $\bar{\varepsilon}$ satisfies $\mathbb{E}[(X - I)\mathbb{I}_{\{X \geq I - \bar{\varepsilon}\}}] = K$ but requires $\mathbb{E}[s(X) - I|X \geq I - \bar{\varepsilon}] \geq K$ in the baseline when $s(\cdot)$ is exogenous. This assumption allows us to focus on failures of financing relationship formation purely driven by the entrepreneur's endogenous information production.

2.6. Investor sophistication and information production

In reality, an insider may dictate the entrepreneur's information production activities or receive additional information besides what the entrepreneur produces. For example, the insider may experiment herself, specify what the entrepreneur must do, or use her proprietary business experience and expertise to predict the market demand or project valuation in future financing rounds. Importantly, the insider can set milestones in the initial contract, in which case the insider conditions the next round of funding on prespecified achievements and accomplishments. We collectively refer to the insider's ability to use such information production technology exogenous to the entrepreneur's design as "investor sophistication."

To best understand IPH, Sections 3 and 4 assume that only the entrepreneur has the relevant skill and expertise to design $(\mathcal{Z}, \pi)$ after raising K. This happens when the lender either has no previous experience on the project or it is too costly for him to extract information (e.g., the firm is located in a hardly accessible location, or the

[12] One can interpret the signal as being either public or private as the receiver's private type. Note that the entrepreneur's experimentation affects both the insider's and outsiders' actions, different from Kolotilin et al. (2017), who examine the case of a single receiver with private types.

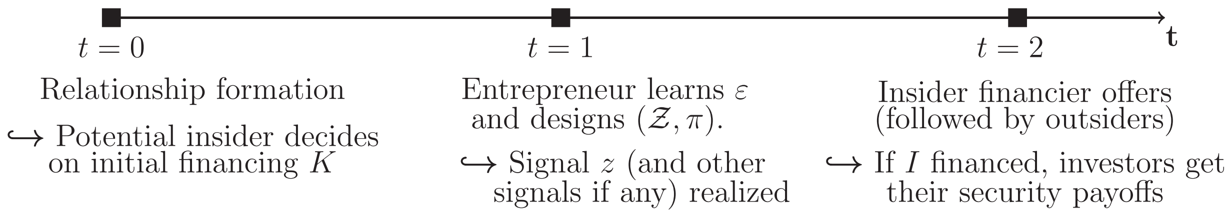

Fig. 1. Timeline of the game.

investor has no relevant expertise to generate independent signals). In Sections 5 and 6.1, we relax the assumption and incorporate investor sophistication by allowing the insider to use a technology to produce an interim signal (not observable to the outsiders) about X.

3. Equilibrium and information production hold-up

To highlight the stark effect of IPH and the way it drastically alters our understanding of relationship finance, we abstract away from contracting in this section and take the security design as exogenous, before allowing contracting at $t = 0$ and highlighting the role of security design in Section 4. Fig. 1 summarizes the timeline of the baseline game.

To characterize the equilibrium, we work backward by first analyzing the interim persuasion game after the formation of the financing relationship. For any given $(\mathcal{Z}, \pi)$, if signal z is privately observed by the insider, then the insider offers $p^I = I$ to finance the project entirely ($\lambda = 1$ endogenously) when $\mathbb{E}[s(X)|z] \geq I$, assuming any indifference (when $\mathbb{E}[s(X)|z] = I$) is resolved by the insider's full financing; otherwise when $\mathbb{E}[s(X)|z] < I$, the insider terminates the project, leading the outsiders to negatively update their valuations and to not invest. The insider's information monopoly essentially gives her full bargaining power over the contractible interim surplus generated, $\max\{\mathbb{E}[s(X) - I|z], 0\}$, which corresponds to the well-known information hold-up in earlier models such as Rajan (1992).

In the case where z is publicly observable (which happens with probability $1-\mu$), then both the insider and the outsiders would offer the competitive $p^I = p^O = \mathbb{E}[s(X)|z]$ when $\mathbb{E}[s(X)|z] - I \geq 0$. The entrepreneur extracts the whole interim surplus in this case.

The entrepreneur's expected payoff for a given realization of ε is thus

$$\begin{aligned} U^E(\mathcal{Z}, \pi; \varepsilon) &= \mathbb{E}[u^E] = \mu \int_0^1 \sum_{z \in \mathcal{Z}^+} (\varepsilon + X - s(X)) \pi(z|X) f(X) dX \\ &\quad + (1-\mu) \int_0^1 \sum_{z \in \mathcal{Z}^+} (\varepsilon + X - s(X) + \mathbb{E}[s(X)|z] - I) \\ &\quad \times \pi(z|X) f(X) dX \\ &= \int_0^1 \sum_{z \in \mathcal{Z}^+} (\varepsilon + X - I) \pi(z|X) f(X) dX \\ &\quad - \mu \int_0^1 \sum_{z \in \mathcal{Z}^+} (s(X) - I) \pi(z|X) f(X) dX, \end{aligned} \tag{5}$$

where $\mathcal{Z}^+ = \{z \in \mathcal{Z} | \mathbb{E}[s(X)|z] \geq I\}$ corresponds to the set of signals inducing continued investment. Eq. (5) follows from applying the law of iterated expectations to $\mathbb{E}[\mathbb{E}[s(X)|z]| z \in \mathcal{Z}^+]$. Correspondingly, the insider's payoff is[13]

$$U^I(\mathcal{Z}, \pi) = \mathbb{E}[u^I] = \mu \int_0^1 \sum_{z \in \mathcal{Z}^+} (s(X) - I) \pi(z|X) f(X) dX. \tag{6}$$

From Eq. (5), the entrepreneur solves the following maximization problem:

$$\max_{(\mathcal{Z}, \pi)} \mathbb{E}[(\varepsilon + X - \mu s(X) - (1-\mu) I) \mathbb{I}_{\{\mathbb{E}[s(X)|z] \geq I\}}]. \tag{7}$$

Proposition 1 (Entrepreneur's optimal experimentation). *The entrepreneur conducts an optimal experiment that entails two signals, i.e., ($|\mathcal{Z}| = 2$). A signal h induces investment if* $X \geq \max\{\bar{X}, \hat{X}(\mu)\}$, *and a signal l induces termination otherwise, where* $\bar{X}$ *and* $\hat{X}(\mu)$ *solve*

$$\mathbb{E}[s(X)|X \geq \bar{X}] - I = 0 \tag{8}$$

$$\varepsilon + \hat{X}(\mu) - \mu s(\hat{X}(\mu)) - (1-\mu) I = 0 \quad \text{if} \quad \varepsilon - (1-\mu) I < 0 \tag{9}$$

$$\hat{X}(\mu) = 0 \quad \text{if} \quad \varepsilon - (1-\mu) I \geq 0. \tag{10}$$

Moreover, all optimal experiments lead to the same investment and payoffs, rendering the equilibrium essentially unique.

Proposition 1 characterizes the optimal experimentation after relationship formation. Eq. (8) indicates that the continuation based on $X \geq \max\{\bar{X}, \hat{X}(\mu)\}$ makes the insider financier at least break even; Eq. (9) indicates that if the private benefit is small relative to the competition, the entrepreneur rationally induces the continuation at X if and only if he can break even; Eq. (10) just states that if the private benefit is large relative to interim competition, the entrepreneur always benefits from the continuation, and the threshold is again pinned down by $\bar{X}$ in Eq. (8).

Note that the threshold structure of the optimal experimentation is consistent with Szydlowski (2020), who derives it for a lumpy investment under any given security

[13] Eqs. (5) and (6) reveal that for a given experiment, the entrepreneur's expected payoff is decreasing and the insider's expected payoff is increasing in μ, a measure of the insider's information monopoly (opposite to competition). By juxtaposing Eqs. (4) and (6), we see that financing relationship is feasible only when the interim rent is sufficiently high (i.e., $\mu \mathbb{E}[(s(X) - I) \mathbb{I}_{\{\mathbb{E}[s(X)|z] \geq I\}}] \geq K$). These observations confirm the results in Petersen and Rajan (1995) that less interim competition leads to the greater possibility of a financing relationship.

design. As will become clear below, our main incremental contributions are to show how competition (captured by $1-\mu$) and the insider's information production affect the experiment's informativeness (characterized by the threshold) as well as how endogenous information production leads to a novel form of hold-up in relationship finance.

In equilibrium, while all profitable projects receive continued financing, some negative NPV (net present value) ones do as well due to the entrepreneur's persuasion. When the insider privately observes the experiment's outcome, she bears the cost of inefficient continuation and the entrepreneur would like to lower the threshold for investment for his private benefit; but as μ decreases, the entrepreneur's chance of getting a competitive price becomes higher, helping him better internalize the cost of inefficient continuation. These trade-offs determine the optimal threshold. In the extreme case, $\mu=0$, the entrepreneur sends a high signal for $X \geq I-\varepsilon$, which induces the socially efficient outcome; at the other extreme, $\mu=1$, the entrepreneur decreases the threshold to make the insider indifferent between investment and termination ($\mathbb{E}[s(X)|z=h]=I$). The proposition reflects the general phenomenon in persuasion games that a sender can "squeeze" a receiver's rent.

Two questions are particularly relevant for relationship finance. First, what does this endogenous information design imply for relationship formation? Corollary 1 reveals that IPH may severely preclude (relationship) financing, both when the insider enjoys a strong information monopoly over the interim signal and when she faces intense interim competition.

Corollary 1 (Information production hold-up). *For any K, information production and investment are socially inefficient for all μ. In particular, there exists $0<\mu^l<\mu^h<1$ such that when $\mu \in [0, \mu^l)\cup(\mu^h, 1]$, relationship financing is infeasible.*

Recall from Section 2.5 that with efficient information production, the financing relationship is feasible and socially efficient. But endogenous information production is inefficient. Obviously, too much interim competition prevents initial investment from potential insiders because they do not accrue enough interim rent to cover the initial investment K. Surprisingly, with full information monopoly, the insider financier could also be held up.[14] Because the entrepreneur produces imperfect information to inefficiently continue projects, the insider's payoff is not monotone in μ. In particular, when the insider enjoys information monopoly (μ approaches 1), her initial investment to form the relationship cannot be recouped. This reverse hold-up is in sharp contrast with the traditional hold-up in an exogenous information setting, in which the possibility of the relationship financing is the highest when $\mu=1$ and the entrepreneur's effort is held up instead.

Given this IPH, the second question pertinent to finance is whether contingent transfers in the form of security payments would solve the problem. Contrasting Corollary 1 with Eqs. (5) and (6), it should also be apparent that taking information production as exogenous in relational financing is not innocuous—whether information production is endogenous determines whether the entrepreneur or the investor has interim bargaining power.

4. Contractual solution for information production hold-up

So far, we have assumed that the entrepreneur only issues securities at $t=1$. In this section, we allow long-term contracts at $t=0$ that specify the security payoffs and the amount of the securities the insider can purchase at $t=1$. In other words, the entrepreneur can contract on λ, the fraction of investment I to be financed from the insider financier in the second round, and the corresponding payment to the insider $s_I(\,\cdot\,)$.[15] Outsiders observe the insider's decision and finance the remaining $(1-\lambda)I$ by purchasing security $s_O(\,\cdot\,)$ at a competitive price p^O. Recall that the security payoffs can depend on X but not on $(\mathcal{Z}, \pi)$ or the interim signals, as described in Section 2.4. Furthermore, similar to the baseline setup, the outcome of the entrepreneur's experimentation is observed privately by the insider with probability μ and publicly by both the insider and outsiders with probability $1-\mu$.

Every contract can then be summarized by a triplet $\{s_I(\,\cdot\,), s_O(\,\cdot\,), \lambda\}$. Because the contract specifies both how the cost I is shared and how the contingent payments depend on future cash flows, it constitutes a general contracting space. Fig. 2 displays the timing of the interactions.

4.1. Security design and long-term contracting

Here we endogenize the security design $s_I(X)$ for the insider and $s_O(X)$ for the outsiders and show that the first-best outcomes are restored. Online Appendix OA2 derives optimal long-term contracts with exogenous security types and shows that IPH is robust to contracting on the verifiable cash flow X.

First, note that the entrepreneur's expected payoff from the contract (s_I, s_O, λ) is then given by

$$U^E=\int_0^{\bar{\varepsilon}}\int_0^1(\varepsilon+X-s_I(X)-s_O(X)+p^O(\varepsilon)-(1-\lambda)I)\mathcal{I}(X;\varepsilon)f(X)g(\varepsilon)dXd\varepsilon, \qquad (11)$$

where p^O is the amount raised from the outsiders by selling the security $s_O(X)$ and $\mathcal{I}$ is the probability of investment at state $X \in [0,1]$, when ε is realized. The outsiders' information set contains the public signal z (if any) they receive with probability $1-\mu$, along with the inference from the insider's action of continuation or termination.

[14] Our results are robust to allowing agents to renegotiate the security after $t=0$ as long as the renegotiation protocols do not depend on agents' posterior beliefs on X (otherwise it is equivalent to contracting on experiment outcome). For any informational environment, the entrepreneur may extract the full interim surplus no matter what the security is. Therefore, the entrepreneur always pools lower types of X with higher ones to make the insider break even.

[15] The insider faces no competition from the outsiders regarding $s_I(\,\cdot\,)$ and thus pays exactly λI upon continuation and getting $s_I(\,\cdot\,)$.

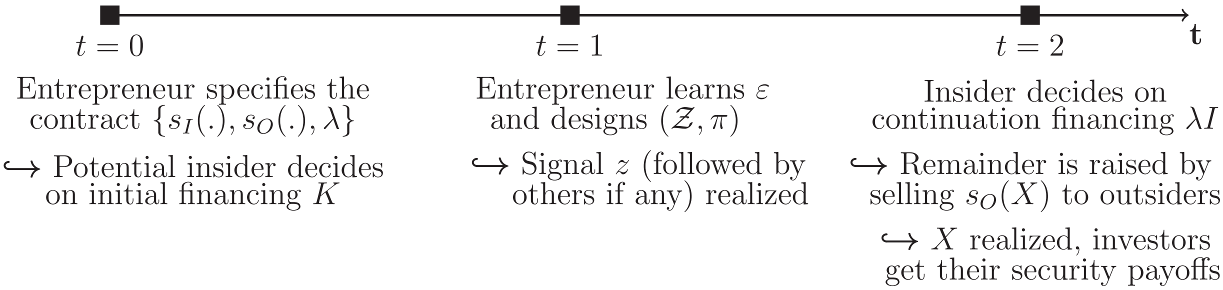

Fig. 2. Timeline of the game with optimal security design.

Outsiders, being competitive, pay a "fair price" p^O given by

$$p^O(\varepsilon) = \frac{1}{\mathbb{E}[\mathcal{I}(X;\varepsilon)]}\int_0^1 s_O(X)\mathcal{I}(X;\varepsilon)f(X)dX. \tag{12}$$

Combining Eq. (11) and (12) we have

$$U^E = \int_0^{\bar{\varepsilon}}\int_0^1 M(X;s_I,s_O,\lambda,\varepsilon)\mathcal{I}(X;\varepsilon)f(X)g(\varepsilon)dXd\varepsilon, \tag{13}$$

where

$$M(X;s_I,s_O,\lambda,\varepsilon) = \varepsilon + X - s_I(X) - (1-\lambda)I. \tag{14}$$

The entrepreneur's optimal contract design then corresponds to the following maximization problem:

$$\begin{aligned} &\max_{s_I(.),s_O(.),\lambda} \ \mathbb{E}[M(X;s_I,s_O,\lambda,\varepsilon)\mathcal{I}^*(X;\varepsilon)] \\ &s.t. \quad \mathbb{E}[(s_I(X)-\lambda I)\mathcal{I}^*(X;\varepsilon)] \\ &\geq K \quad and \quad s_I(X)+s_O(X)\leq X \qquad \forall X\in[0,1], \end{aligned} \tag{15}$$

where the optimization is over the set of designs and the option to walk away from the financing relationship and the constraints are the incentive-compatibility condition of the insider to form a relationship and the entrepreneur's limited liability. $\mathcal{I}^*(\cdot;\varepsilon)$ is the equilibrium investment function under the optimal experiment $(\mathcal{Z}^*(\varepsilon),\pi^*(\varepsilon))$ and is given by

$$\mathcal{I}^*(X;\varepsilon) = \sum_{z\in\mathcal{Z}^*(\varepsilon)}\pi^*(z|X,\varepsilon)\mathbb{I}_{\{\mathbb{E}[s_I(X)-\lambda I|z]\geq 0\}\cap\{\mathbb{E}[s_O(X)-(1-\lambda)I|\mathcal{F}^O]\geq 0\}}, \tag{16}$$

and the optimal experiment given the contract $\{s_I(\cdot), s_O(\cdot), \lambda\}$ and ε solves the following:

$$\begin{aligned} &\max_{(\mathcal{Z},\pi)}\int_0^1 M(X;s_I,s_O,\lambda,\varepsilon)\mathcal{I}(X;\varepsilon)f(X)dX \\ &\quad \text{where} \quad \mathcal{I}(X;\varepsilon) \\ &= \sum_{z\in\mathcal{Z}^*(\varepsilon)}\pi(z|X,\varepsilon)\mathbb{I}_{\{\mathbb{E}[s_I(X)-\lambda I|z]\geq 0\}\cap\{\mathbb{E}[s_O(X)-(1-\lambda)I|\mathcal{F}^O]\geq 0\}}. \end{aligned} \tag{17}$$

4.2. Optimal solutions

The contracting problem involves infinitely dimensional nested optimization, and the solution methodologies in conventional contracting and security design problems do not easily apply. We instead take a constructive proof approach by conjecturing the optimal designs and show that this set of designs uniquely achieve the first-best outcome and are indeed optimal in the sense that they maximize the entrepreneur's ex-ante payoff for all ε.

Proposition 2 (Optimal design). *An optimal contract exists and implements the first-best social outcome. All optimal contracts induce experiments that generate a continuation if and only if* $X \geq I-\varepsilon$. *Moreover, they essentially all involve the use of convertible securities and are described by* $\{s_I(\cdot), s_O(\cdot), \lambda\}$, *satisfying the following conditions:*

$$\lambda \in \left[0, \frac{I-\bar{\varepsilon}}{I}\right] \tag{18}$$

$$s_I(X) = \min\{\lambda I, X\}, \qquad \forall\, X < I \tag{19}$$

$$\mathbb{E}[(s_I(X)-\lambda I)\mathbb{I}_{\{X\geq I\}}] = K \tag{20}$$

$$s_O(X) = X - s_I(X), \qquad \forall\, X \in [0,1]. \tag{21}$$

Eq. (18) reflects the partial indeterminacy of the optimal design because λ can take on a range of values. Eq. (19) requires that in the bad states of the world, the security is debt-like. In fact, the shape of the securities in the region $X < I-\bar{\varepsilon}$ is indeterminate, but in terms of payoffs, they are equivalent, thus the use of word "essentially" in the proposition. Eq. (20) ensures that the insider breaks even ex ante but leaves the security shape indeterminate. Notice that when we achieve the first-best social outcome, the insider gets paid only when the project is continued $X \geq I-\varepsilon$. But the insider's payoff is zero for $X < I$ anyway because ε is a $t=0$ random variable whose realizations are assumed to be noncontractible. Therefore, the indicator function in Eq. (20) is independent of ε. Finally, Eq. (21) is driven by the entrepreneur's limited liability: as we argue shortly, she can use outsiders to commit herself to internalizing the cost inefficient continuation but still cannot give outsiders more than $X - s_I(X)$.

Proposition 2 essentially states that entrepreneurs optimally use convertible securities for insiders and residuals for outsiders.[16] Fig. 3 provides a concrete illustration of

[16] In Online Appendix OA3, we show equity or debt could be optimal when the entrepreneur is constrained to issue the same security for the insider and the outsiders ($s_I(X) = \lambda s(X)$, $s_O(X) = (1-\lambda)s(X)$, for some $s(X)$) due to regulatory reasons. In Online Appendix OA4, we discuss how our contractual setting relates to the classical literature on contracting and moral hazard.

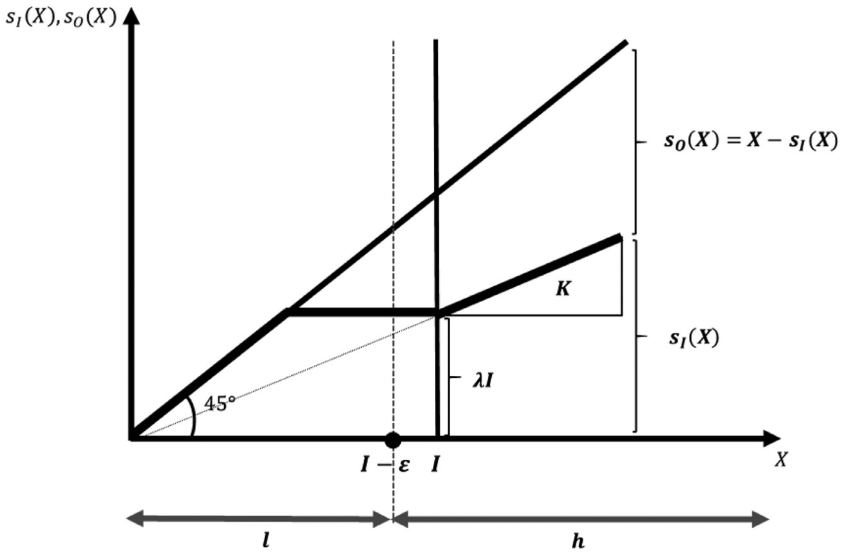

Fig. 3. Illustration of optimal securities under flexible design.

the optimal contract using convertible notes for the insider and equities for the outsiders. The dotted line indicates the threshold for continuation versus termination signals.

We now provide the intuition for Proposition 2. Because the inefficiency lies in the continuation of bad projects ($X + \varepsilon < I$), what matters is how the security makes the information producer (the entrepreneur) internalize the cost of such inefficiency (partially through outsiders that are present). From the insider's perspective, efficient continuation then entails her paying the fair value for the security λI when the investment is socially marginal (zero NPV), lest there is either a form of debt overhang resulting in underinvestment (when $s_I(I - \varepsilon) > \lambda I$ and the insider gets more than her fair share of the surplus) or a subsidy from the insider that leads to overinvestment (when $s_I(I - \varepsilon) < \lambda I$).

At first, it seems that an optimal security only requires $s_I(I - \varepsilon) = \lambda I$, which makes the entrepreneur (i) indifferent between continuing and not continuing at $X = I - \varepsilon$ (social NPV is zero), (ii) strictly preferring continuation when $X > I - \varepsilon$, and (iii) strictly preferring termination when $X < I - \varepsilon$. Note that (ii) and (iii) follow from the monotonicity of $M(X;\, s_I,\, s_O,\, \lambda,\, \varepsilon)$ in X. However, ε represents an uncertainty resolved at $t = 1$, which then affects the entrepreneur's information production, and the entrepreneur cannot design the security at $t = 0$ to be dependent on one particular value of ε.[17]

As a result, committing to a large enough $1 - \lambda$ to expose the entrepreneur's payoff to inefficient continuation for the entire region of $X = I - \varepsilon,\ \varepsilon \in (0, \bar{\varepsilon})$ yields the debt-like flat region in Fig. 3.[18] Meanwhile, the security design also needs to ensure that the insider earns enough from the second round to cover the initial investment K. This restricts the security somewhat, but as long as the area under s_I, but above the horizontal line λI, reaches K, its shape to the right of the debt-like region is indeterminate. Nevertheless, the endogenous experimentation leads to a determinate informational environment and investment decision that is also socially optimal.

We have demonstrated that optimal designs have to entail some form of debt-likeness and convertibility, consistent with real practice. Note that the debt-likeness is not driven by risk-bearing capacity or informational asymmetry but by the entrepreneur's bias for continuation and contract incompleteness on information production. It aligns the entrepreneur's incentives in information production with that of the social planner and his ex-ante self.

Overall, Proposition 2 indicates that (i) conclusions from earlier studies are robust to introducing endogenous and flexible information production and (ii) the optimal security for the insider not only has a debt-like region but can also rationalize the use of a large class of securities in real life, an empirical observation other models do not fully account for.[19]

[17] As we discuss in Online Appendix OA1, this is a general phenomenon when the information design space (from experimentation) differs from the contracting space (which is based on the cash flow only), not an artifact of our baseline assumption on ε.

[18] We prove in Online Appendix OA1 that even with a deterministic ε at $t = 0$, an optimal security design entails this debt-like flat region as long as there is some uncertainty at $t = 0$ (about the misalignment or final project cash flow) that resolves through the course of the financing relationship (starting from $t = 1$) because then pinning down a single point of the security based on ε does not trivially correct the divergence of incentives.

[19] Our goal is not to introduce an alternative mechanism or competing theory for the predominant use of convertible securities in venture capital (Kaplan and Strömberg, 2003) or to proclaim the information design channel to be the most dominant. Besides proving the optimality of convertible securities, we emphasize the role of arm's-length outsiders and the need for the joint optimal security design for both the insider and outsiders, which extant theories do not discuss (Inderst and Vladimirov, 2019 is a notable exception).

5. Relationship finance under investor sophistication

In this section, we show how investor sophistication—the insider's ability to use a given information production technology other than the entrepreneur's experimentation to generate interim information—mitigates IPH. Interestingly, its interaction with interim competition helps us rationalize puzzling empirical patterns in the formation of lending relationships. We also prove that the contractual solution in Proposition 2 is robust to the level of the insider's sophistication. By doing so, we essentially develop the solution for a Bayesian persuasion game involving multiple receivers facing discriminatory disclosures. Finally, we discuss contracting on interim events (milestones).

We incorporate the insider's information production by endowing her with an exogenous technology to produce a proprietary signal about X during the financing relationship. In particular, the insider uses an experiment $(\mathcal{Y}, \omega_q)$, where $\mathcal{Y} = \{y_1, y_2, \ldots, y_m\}$ is a finite set with m signals. $\omega_q(y_i|X)$ represents conditional distributions, and we assume the posterior distribution induced by an experiment to be atomless with a full support. $q \in [0, 1]$ is an index we employ to rank the informativeness of different experiments, with $q = 0$ for an uninformative experiment. For $q > q' \in [0, 1]$, the experiment $(\mathcal{Y}, \omega_q)$ is more informative than $(\mathcal{Y}, \omega_{q'})$ in the sense of Blackwell (1953).

Moreover, for every q, we assume the signals in $\mathcal{Y}$ can be ranked: for every $m \geq i > i' \geq 1$, distribution $f(X|y_i)$ dominates distribution $f(X|y_{i'})$ in the sense of first-order stochastic dominance (i.e., for every $X \in (0, 1)$, we have $F(X|y_i) < F(X|y_{i'})$). This assumption directly implies that $\mathbb{E}_q[s(X)|y_i, z] > \mathbb{E}_q[s(X)|y_{i'}, z]$ for every security $s(\cdot)$ and signal z with a nondegenerate distribution. The following example illustrates the points above.

Example 1. The insider's experiment generates a binary signal, i.e., $\mathcal{Y} = \{\tilde{h}, \tilde{l}\}$, with the following information structure for $(\{\tilde{h}, \tilde{l}\}, \omega_q)$:

$$\omega_q(\tilde{h}|X) = \begin{cases} \frac{1+q}{2} & I \leq X \leq 1 \\ \frac{1-q}{2} & 0 \leq X < I. \end{cases} \quad (22)$$

The investor receives a signal $y = \tilde{h}$ with probability $\frac{1+q}{2} \geq \frac{1}{2}$ if the project is profitable and $y = \tilde{l}$ with probability $\frac{1-q}{2} \leq \frac{1}{2}$ otherwise. Clearly $\mathbb{E}_q[s(X)|\tilde{h}] \geq \mathbb{E}_q[s(X)|\tilde{l}]$ for every $q > 0$ and security $s(\cdot)$. Moreover, if $1 > q > q' > \frac{1}{2}$, then the following inequalities also hold:

$$\mathbb{E}_q[s(X)|\tilde{h}] > \mathbb{E}_{q'}[s(X)|\tilde{h}] > \mathbb{E}_{q'}[s(X)|\tilde{l}] > \mathbb{E}_q[s(X)|\tilde{l}]. \quad (23)$$

The inequality means that experiments with higher values of q generate relatively more extreme signals.

Given the insider's information technology, the entrepreneur now designs an experiment $(\mathcal{Z} \times \mathcal{Y}, \pi)$, where $\pi(.,.|X) : \mathcal{Z} \times \mathcal{Y} \to [0, 1]$ is the joint conditional probability of observing the signals. Note that the marginal distributions for $y \in \mathcal{Y}$ must be consistent with the insider's experiment, (i.e., $\sum_{z \in \mathcal{Z}} \pi(z, y|X) = \omega_q(y|X)$ for every $X \in [0, 1]$ and $y \in \mathcal{Y}$). Moreover, we allow the signals in $\mathcal{Z}$ and $\mathcal{Y}$ to be correlated conditional on the true state of the world. Similar to the baseline case, the outsiders observe signal z with probability $1 - \mu$, while signal y is always privately observed by the insider financier.

5.1. Information production with investor sophistication

We start by characterizing in Lemma 1 the optimal experiment under investor sophistication. We denote the signal indicating whether $X \geq \max\{\hat{X}(\mu), \bar{X}(y_i)\}$ by x^i_H, where $\bar{X}(y_i)$ is the solution to $\mathbb{E}_q[s(X) - I|y_i, X \geq \bar{X}(y_i)] = 0$ if a solution exists and is zero otherwise; we denote the opposite signal indicating whether $X < \max\{\hat{X}(\mu), \bar{X}(y_i)\}$ by x^i_L.

Lemma 1 (Endogenous information under investor sophistication). *An optimal experiment exists and requires at most $2|\mathcal{Y}| = 2m$ signals. For every signal $y_i \in \mathcal{Y}$, the entrepreneur sends either $z^h_i = \{y_i, x^i_H\}$ or $z^l_i = \{y_i, x^i_L\}$.*

Lemma 1 implies that the entrepreneur can split the information design problem into m separate problems, each characterized by Proposition 1.[20] Intuitively, the entrepreneur prefers to reveal y to the outsiders to level the playing field by informing them of y and by eliminating the insider's informational advantage. Since the optimal experiment in Proposition 1 has at most two signals, the optimal experiment in the presence of a sophisticated investor has at most $2m$ signals.

Proposition 3. (a) For every $\mu \in [0, 1]$, the insider's expected payoff $U^I(\mu; q)$ is weakly increasing in q.

(b) For any given $(\mathcal{Y}, \omega_q)$, the equilibrium investment decision is not ex-post socially optimal with a positive probability.

Part (a) in Proposition 3 states that the insider's interim payoff increases with the informativeness of his endowed signal, which follows from Lemma 1 and Blackwell's theorem (Blackwell, 1953).[21] Part (b) derives from combining Corollary 1 with Lemma 1. The entrepreneur still tends to induce overinvestment, as he does not internalize the cost of experimentation K. Moreover, even in the case that the insider's signal is highly informative, the insider tends to underinvest since she does not internalize the entrepreneur's private benefit from the investment.

Overall, the proof further reveals that, while they improve the entrepreneur's information production, the first-best outcomes cannot be achieved through investor sophistication alone. As such, initial long-term contracts with the right security design are integral to achieving the socially optimal investment, as we discuss in Section 5.3.

5.2. Relationship formation, sophistication, and competition

The theoretical predictions on the effect of competition on bank orientation have been ambiguous. According to

[20] A priori, it is not obvious that the entrepreneur should design the experiment to correspond with y type by type. For instance, if the insider and outsiders bid for the security simultaneously, the entrepreneur might find it optimal to only partially disclose the insider's signal to the outsiders. The simple design follows only after taking into consideration that outsiders learn from the insider's interim action.

[21] Both Kolotilin (2018) and Guo and Shmaya (2019) derive nonmonotone results for the case that the sender's and the receiver's experiments must generate independent signals conditional on X. We differ in that we do not restrict the insider's and the entrepreneur's information production to be independent. In Online Appendix OA5, we illustrate how our results extend to the setting with such independence requirement.

the investment theory (e.g., Petersen and Rajan, 1995; Dell'Ariccia and Marquez, 2004), as the credit market concentration decreases, firms borrowing options expand, rendering banks less capable of recouping their initial investments during the lending relationship, which hinders relationship banking. According to the strategic theory (e.g., Boot and Thakor, 2000; Dinc, 2000), fiercer interbank competition drives local lenders to take advantage of their competitive edge and to reorient lending activities toward relational-based lending to small local firms, which strengthens relationship banking. Others (e.g., Yafeh and Yosha, 2001; Anand and Galetovic, 2006) suggest that competition can have ambiguous effects on lending relationships but typically predict an inverted U-shaped pattern. Yet empirically, Elsas (2005) and Degryse and Ongena (2007) show a U-shaped relationship between the likelihood of a lending relationship and the level of competition in the credit market.[22] Proposition 4 offers an explanation.

Proposition 4 (Relationship and competition). $\exists\ \underline{\mu}(q) \in (0, 1)$ *such that for* $\mu \in [\underline{\mu}(q), 1]$, *the insider's payoff from the relationship financing,* $U^I(\mu;\ q)$, *is increasing in the level of interim competition* $(1-\mu)$ *for unsophisticated investors* $(q=0)$, *decreasing for sophisticated investors (sufficiently large q), and U-shaped for investors with intermediate sophistication.*

On the one hand, for a fixed level of private benefit of continuation, lower levels of competition increase the insider's share of the surplus and are preferred by more sophisticated investors who can produce their own information. On the other hand, higher levels of competition can encourage more efficient information production from the entrepreneur, which increases total surplus. Thus, it is preferred by the less sophisticated investors who have no other means of obtaining information rent. For intermediate values of sophistication, competition hurts the insider's profit until it replaces the investor's independent information as her main source of interim rent, leading to the local U-shape.

Fig. 4 illustrates the relationship between $U^I(\mu;\ q)$ and μ, the inverse measure of competition in our model. In particular, when q takes intermediate values and μ is exogenous, our model thus helps rationalize the findings of Elsas (2005) and Degryse and Ongena (2007). As is expected and shown in Fig. 4, relationship formation eventually decreases with competition when the market becomes extremely competitive (μ gets closer to zero).

5.3. Optimal contracting and security design

Next, we examine whether security design and contracting at $t=0$ fully mitigate IPH under investor sophistication. Proposition 5 shows that Proposition 2 is robust to insider sophistication and still achieves the first-best outcome.

Proposition 5 (Optimal contracts with investor sophistication). *Regardless of what* $(\mathcal{Y}, \omega_q)$ *the insider is endowed with, all optimal securities are characterized by Eqs. (19)–(21).*

The intuition behind Proposition 5 is the following: because of the flat part of the security, the insider's security entails no risk when the entrepreneur reveals that $X > I - \varepsilon$. Therefore, the insider always continues the project when $z = h$, regardless of her own signal. In other words, the insider's action only reveals whether $X > I - \varepsilon$, and consequently, the outsiders invest if and only if the insider invests.

In contrast to the case without long-term contracts (Lemma 1), the entrepreneur's experiment under the optimal long-term contract does not depend on the insider's information production. In other words, the entrepreneur might find it optimal to hide some of the insider's information from the outsiders. Consequently, information asymmetry exists between the insiders and the outsiders, but it does not prevent implementation of the socially optimal investment, because the contractual solution resolves the inefficiencies.

5.4. Setting milestones

Contracting on interim events is related to "milestones" used in venture financing. For example, the entrepreneur can commit to reaching a prespecified customer base before seeking additional financing. Would that solve the IPH problem? Perhaps surprisingly, the insider cannot increase her expected payoff by contracting on interim events.

Note that contracting on milestones is different from contracting on $(\mathcal{Z}, \pi)$. We use $\mathcal{Y}^b \subset \mathcal{Y}$ to denote binding signals, following which the insider commits to either continue or terminate financing. We denote the set of nonbinding signals by $\mathcal{Y}^{nb} = \mathcal{Y} \setminus \mathcal{Y}^b$.

Corollary 2 (Milestone futility). *The insider cannot gain from setting milestones. In particular, the insider's expected payoff is maximized when* $\mathcal{Y}^b = \emptyset$.

Corollary 2 helps explain why milestones are seldom binding in practice and early stage projections are rarely made or enforced. The entrepreneur can flexibly control how the insider updates her prior for every $y \in \mathcal{Y}^{nb}$, regardless of the choice of $\mathcal{Y}^b$. Therefore, the insider's information set following signals in $\mathcal{Y}^{nb}$ does not change, while she potentially makes suboptimal decisions following signals in $\mathcal{Y}^b$ due to the binding commitment.

6. Discussions and extensions

Our main findings are robust under alternative model specifications. In the Online Appendix, we discuss (i) IPH and security design under restricted experimentation space (OA1); (ii) contracting and security design under IPH when the entrepreneur can only use one type of security (OA3); (iii) how IPH can also reduce the effort distortion introduced by Rajan (1992) (OA6); (iv) what happens when

[22] These two studies stand out because they measure relationship banking directly in terms of duration and scope of interactions, thus improving upon indirect measures such as the loan rate (Petersen and Rajan, 1995) or credit availability over firms' lifetimes (Black and Strahan, 2002), for which the impact of competition could be ambiguous in equilibrium (Boot and Thakor, 2000).

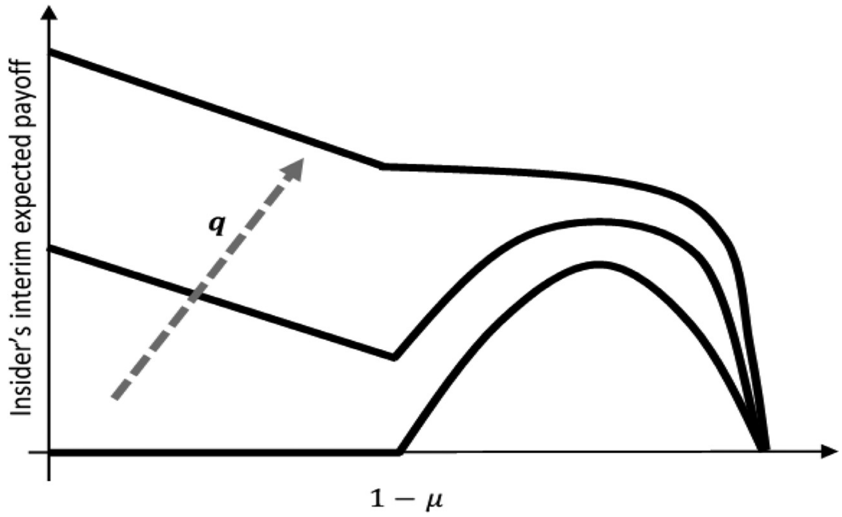

Fig. 4. Illustration of the equilibrium capacity of the financing in the initial round as a function of the level of ex-post competition.

we allow a continuum of actions, i.e., scalable projects (OA7); and (v) the case of allocating security design right to the insider (OA8).

We next discuss how costly and endogenous information production or acquisition by the insider affects the entrepreneur's information production (Section 6.1) as well as the roles of the commitment to, and the observability of, experimentation (Section 6.2), both of which are fundamental issues in the information design literature.

6.1. Costly information production by the insider

In our baseline setup (Section 3), only the entrepreneur has the expertise to experiment. Although in Section 5 we endow the insider with a specific information production technology, one may question what happens if the insider can also endogenously produce or acquire information. We now show that the insider's endogenous and costly experiment improves the entrepreneur's information production and consequently facilitates the relationship finance. Nevertheless, it does not fully resolve IPH, and the information production is still socially inefficient.

Specifically, suppose an insider can pay $c > 0$ to observe X.[23] For illustration and simplicity, we assume $\mu = 1$, (i.e., the insider faces no interim competition). Then following the realization of z, the insider incurs the cost and learns X if and only if the downside risk implied by the signal is large enough:

$$\mathbb{E}[\{s(X) - I\}^{+}|z] - c \geq \mathbb{E}[s(X) - I|z] \iff -c \geq \mathbb{E}[\{s(X) - I\}^{-}|z], \tag{24}$$

where $\{x\}^{+} \equiv \max\{x, 0\}$ and $\{x\}^{-} \equiv \min\{x, 0\}$.

Therefore, the entrepreneur, aiming to maximize the probability of the investment, should choose a threshold larger than the baseline threshold $\bar{X}$ when c is sufficiently small:

$$-c > \mathbb{E}[\{s(X) - I\}^{-}|X \geq \bar{X}]. \tag{25}$$

In this case, the entrepreneur's optimal threshold is $\bar{X}^c > \bar{X}$, which satisfies

$$-c = \mathbb{E}[\{s(X) - I\}^{-}|X \geq \bar{X}^c]. \tag{26}$$

We see that the insider's access to the costly information production technology can discipline the entrepreneur's experimentation by imposing an implicit constraint on the set of the entrepreneur's experiments (Eq. (24) in our setting). The constraint typically alleviates IPH and generally depends on the specific information production technology available to the investors.

Eq. (26) also provides insights about which types of costly signals and security forms discipline the entrepreneur's experimentation more. As for the former, suppose the insider has access to all potential threshold experiments by paying c (i.e., she learns whether $X < T$ for a threshold level $T \in [0, 1]$ she prespecifies). By repeating the arguments above, we see that a less informative signal induces the same entrepreneur's experiment, with the same threshold $\bar{X}^c$. Therefore, what really matters for the entrepreneur's information production is how cheaply the insider can assess the project's downside risks, which lies at the core of the misalignment of interests.

Regarding the role of the form of the security $s(\cdot)$, according to Eq. (24), securities that allocate more downside risk to the insider are more effective in disciplining the entrepreneur. Corollary 3 formalizes this point.

Corollary 3. For securities $s^1(\cdot)$ and $s^2(\cdot)$, suppose $s^1(X) - I$ and $s^2(X) - I$ are negative over the same range of values $[0, X^s]$, for some $X^s \in (0, 1)$, and the following condition holds:

$$s^1(X) - I \leq s^2(X) - I \leq 0 \qquad \forall\ X \leq X^s. \tag{27}$$

Then the entrepreneur chooses experiments with a threshold structure under both securities; however, the threshold

[23] Obviously, the insider would fully learn X if $c = 0$ and the problem becomes trivial. $c > 0$ reflects the fact that entrepreneur still produces information more efficiently than the investors.

is larger for $s^1(\cdot)$ than $s^2(\cdot)$. In other words, the entrepreneur's experiment induces a more socially efficient investment with security $s^2(\cdot)$ than with security $s^1(\cdot)$.

The intuition for Corollary 3 is that the insider is more likely to acquire the costly signal when she faces a larger potential loss. Consequently, a security that allocates more downside risk to the insider forces the entrepreneur to choose a more informative experiment, to crowd out the insider's information production, in line with the finding of Yang and Zeng (2018) about the prevalent use of equity.

To summarize, we see the insider's access to costly information production technology improves the entrepreneur's information production and increases the possibility of relationship financing. Furthermore, the technology is more effective when the insider is more exposed to potential losses as well as when the technology delivers a more precise assessment of the downside risks. The results here complement those in Section 5 by explaining the role of insider's flexibility in information production instead of the precision of her endowed signals. Both lead to more socially efficient investment outcomes, while none of them fully resolves the IPH issue.

6.2. Commitment to information design and partial observation

Commitment issues are common in information design applications. Our setup does not require that the entrepreneur commit to an experiment when raising K but allows the investor to observe the experiment after becoming an insider. In fact, the lack of commitment and contractibility of the experimentation at $t = 0$ in our setting (partially) breaks the indeterminacy in Szydlowski (2020) and prompts the debt-like region of the optimal security design as a response. As shown above, the choice of security ex ante affects the choice of information design ex post.

Additionally, in the baseline model, we assume that not only does the investor's monitoring technology rule out misreporting, but the investor also commits to perfectly monitoring the entrepreneur's experiment, even though she might be better off randomizing between monitoring and not monitoring. We relax these assumptions and find that while the first-best outcome becomes no longer achievable, our results on IPH and optimal contracting and security design still hold. Specifically, we assume

1. With probability $\alpha \in [0, 1]$, the entrepreneur can misreport the signal z without getting caught, even while being monitored by the investor.
2. At time $t = 0$, the investor commits to verifying the signal realization with probability $\beta \in [0, 1]$.[24]

Note that our baseline model corresponds to $\alpha = 0$ and $\beta = 1$. In general, the following variant of Corollary 1 and Proposition 2 holds.

Proposition 6 (Partial observation and commitment).

(a) *Absent long-term contracts, the insider receives no interim rent for extreme values of μ regardless of the values of $\alpha, \beta \in [0, 1]$.*
(b) *For $\alpha > 0$ and $\beta < 1$, there exists no contractual solution that implements the first-best outcome. For small enough values of α and large enough values of β, the convertible securities are still optimal for all ε, provided an optimal contract exists. Relationship financing is infeasible for large enough values of α and low enough values of β.*

Proposition 6 validates our prior knowledge that the insider's monitoring is essential to relationship financing. But part (a) also shows that the IPH is robust to partial monitoring when the competition is too low or too high. Furthermore, note that in contrast to costly state verification models, where the insider optimally randomizes between monitoring and not monitoring, part (b) shows that the full observation by the investor is required to implement the socially optimal outcome. The main difference is that the investor has no way to castigate the entrepreneur for misreporting, as the latter is protected by his limited liability.

7. Conclusion

We model the dynamic financing of projects by relationship and arm's-length investors as a mechanism design problem with an embedded Bayesian persuasion game whereby the entrepreneur endogenously produces interim information to seek continued financing. We show that the entrepreneur's (sender's) endogenous experimentation typically reduces the insider investor's (receiver's) information monopoly rent, holding up that relationship financier's incentives to form the relationship in the first place. Investor sophistication and interim competition can mitigate the problem, and they interact to produce nonmonotone patterns of relationship formation and interim competition. We then derive optimal sequential securities to resolve the Information Production Hold-up (IPH) problem: the entrepreneur contracts with investors in the initial round to allow them to purchase convertible securities in a later round and issues residual claims to competitive outsiders later.

Our theory broadly applies to hold-up problems in persuasion games with contingent transfers and multiple receivers of different types. It is immediately relevant for at least two major areas of finance. First, it reveals the impact of endogenous information production in relationship lending and clarifies its interactions with investor sophistication and competition, rationalizing the puzzling U-shaped link between bank orientation and interim competition documented. Second, it highlights how IPH can rationalize the use of a large variety of convertible securities in venture capital. Given that the solutions to many of the world's biggest problems, such as Alzheimer's disease, global warming, and fossil fuel depletion, require

[24] Note that if the investor monitors the experiment, she makes the continuation decision based on both the reported and the monitored signal. In this case, we allow the investor to commit to punishing the entrepreneur by not investing in the case of misreporting. We can easily allow the verification to incur a small cost, and the extension would share the spirit of, for example, Trigilia (2019). The key difference from the literature on costly state verification (CSV) is again the endogenous design of the information.

initial funding for experimentation and reliable financing relationships, the cost of inefficient information production could be tremendous. Our study constitutes the first attempt to underscore, formalize, and then examine potential contractual solutions for this practical issue.

Appendix A. Proofs of lemmas and propositions

A1. A technical lemma

The entrepreneur endogenously designs the experiment to maximize his payoff, subject to the insiders' second-round participation constraint. With a finite state space, the signal space as the range of a deterministic mapping from the state space is necessarily finite (Bergemann and Morris, 2016). Consequently, we can apply the method of Lagrange multipliers directly. But alas, we are dealing with infinite dimensional state space and unrestricted signal generation space. Technically speaking, there is no guarantee that one can apply the method of Lagrange multipliers without additional regularity conditions.

We tackle this issue in two parts in Lemma A1. First, we show for any experiment, there exists a binary experiment that yields the entrepreneur the same expected payoff. Therefore, without loss of generality, we can restrict our attention to experiments generating binary signals of continuation versus termination. Had we assumed $\mathcal{Z}$ to be a continuum, our discussions remain the same except that notation-wise we have to mix summations with integrals and allow delta functions in probability densities. Because the experiments we look at are conditional probabilities mappings from the state space to [0,1], they live in a Banach space. With this insight and part (a) of the lemma, we prove a mathematical result in part (b) that allows us to use the method of Lagrange multipliers in the proofs of our lemmas and propositions (see also Ito, 2016 for an abstract generalization).

Lemma (A1). (a) Consider a Bayesian persuasion game à la Kamenica and Gentzkow (2011), with metric state space Ω, ex-ante probability measure μ_Ω, compact metric signal space S, with induced probability measure μ_S, and receiver's action space $a \in \{0, 1\}$. Suppose the sender's and receiver's payoff from action a in space $w \in \Omega$ are $a \times u(w)$ and $a \times v(w)$, respectively, for some real-valued measurable functions $u, v : \Omega \to \Re$. Then, for any experiment, denoted by measurable conditional probability functions $\pi(s|w)$ over S, there exists binary experiment $(\{h, l\}, \pi')$ that implements the same mapping from the states to actions and thus the same expected payoff for both sender and receiver.

(b) Suppose $w_i(x), m_i(x) : [0, 1] \to \Re$ $(1 \le i \le N)$ are continuous and bounded functions. Suppose the following maximization problem has a solution:

$$\max_{\alpha_i(.)\in\mathcal{A}} \int_0^1 \sum_{i=1}^N w_i(x)\alpha_i(x)dx$$
$$s.t. \quad \int_0^1 m_i(x)\alpha_i(x)dx \ge 0 \qquad \forall \;\; 1 \le i \le N,$$
$$and \quad \sum_{i=1}^N \alpha_i(x) \le 1 \qquad \forall x \in [0, 1], \tag{28}$$

where $\mathcal{A}$ is the set of all measurable functions over [0,1] that take value from [0,1]. Then, there exist non-negative real numbers $\{\mu_i\}_{i=1}^N$ such that the solution to Eq. (28) is a solution to the following maximization problem:

$$\max_{\alpha_i(.)\in\mathcal{A}} \quad \int_0^1 \sum_{i=1}^N (w_i(x) + \mu_i m_i(x))\alpha_i(x)dx$$
$$s.t. \quad \sum_{i=1}^N \alpha_i(x) \le 1 \qquad \forall x \in [0, 1]. \tag{29}$$

Proof. Proof of part (a)

Let S^+ be the set of signals that implement action $a = 1$. Therefore, signal s belongs to S^+ if the following condition holds (assuming the ties are broken in favor of action $a = 1$):

$$\int_\Omega v(w)\pi(s|w)d\mu_\Omega \ge 0. \tag{30}$$

Note that S^+ is a measurable subset of S because it is a super level set of function $\int_\Omega v(w)\pi(s|w)d\mu_\Omega$. That said, define experiment $(\{h, l\}, \pi')$ as follows:

$$\pi'(h|w) = \int_{S^+} \pi(s|w)d\mu_S$$
$$and \quad \pi'(l|w) = 1 - \pi'(h|w) \qquad \forall w \in \Omega. \tag{31}$$

In experiment $(\{h, l\}, \pi')$, only signal h induces action $a = 1$ because

$$\int_\Omega v(w)\pi'(h|w)d\mu_\Omega = \int_\Omega \int_{S^+} v(w)\pi(s|w)d\mu_S d\mu_\Omega \ge 0,$$
$$\int_\Omega v(w)\pi'(l|w)d\mu_\Omega = \int_\Omega \int_{S\backslash S^+} v(w)\pi(s|w)d\mu_S d\mu_\Omega < 0. \tag{32}$$

Eq. (33) shows for any $w \in \Omega$, the unconditional probabilities of action $a = 1$ taken by the sender in experiments (S, π) and $(\{h, l\}, \pi')$ are equal. That directly implies that the expected payoff of the sender and receiver are also the same under these two experiments. It completes the proof.

$$Prob(a = 1|w; (S, \pi)) = \int_{S^+} \pi(s|w)d\mu_S = \pi(h|w)$$
$$= Prob(a = 1|w; (\{h, l\}, \pi')). \tag{33}$$

Proof of Part (b)

Let $\tilde{\mathcal{A}}^N$ be the set of all $N-$tuples of functions $(\alpha_1(.), \ldots, \alpha_N(.))$ in $\mathcal{A}$ that satisfy $\sum_{i=1}^N \alpha_i(x) \le 1$, for every $x \in [0, 1]$.

Since all functions are bounded and measurable, it is easy to check that $\tilde{\mathcal{A}}^N$ constitutes a closed set in $\mathcal{L}^{1^N}$. The following maximization problem is then well-defined.

$$\max_{(\alpha_1(.),\ldots,\alpha_N(.))\in\tilde{\mathcal{A}}^N} \int_0^1 \sum_{i=1}^N w_i(x)\alpha_i(x)dx$$
$$s.t. \quad \int_0^1 m_i(x)\alpha_i(x)dx \ge 0 \qquad \forall \;\; 1 \le i \le N. \tag{34}$$

Suppose $a^* \in \tilde{A}^N$ is the solution to the problems (28) and (34). It is easy to see that the Slater condition and strong duality hold. Therefore, there exists a vector of nonnegative real numbers $\{\mu_i\}_{i=1}^N$ such that a^* also solves the following maximization problem:

$$\max_{(\alpha_1(.),\ldots,\alpha_N(.))\in\bar{\mathcal{A}}^N} \int_0^1 \sum_{i=1}^N w_i(x)\alpha_i(x)dx + \sum_{i=1}^N \mu_i \int_0^1 m_i(x)\alpha_i(x)dx. \quad (35)$$

Note that Eq. (29) is equivalent to Eq. (35), which completes the proof. □

A2. Proof of Proposition 1

Optimality of a binary experiment with threshold structure. According to Eqs. (5) and (6), we can appeal to Lemma A1(a) by considering state space [0,1] and the following utility functions:

$$u(X) \equiv \varepsilon + X - \mu s(X) - (1-\mu)I$$
$$v(X) \equiv \mu(s(X) - I). \quad (36)$$

Therefore, the entrepreneur optimally chooses a binary experiment. Then, employing Lemma A1(b) for $N=1$, the entrepreneur solves the following optimization problem:

$$\max_{\pi(h|X)} \int_0^1 [\varepsilon + X - \mu s(X) - (1-\mu)I]\pi(h|X)f(X)dX \quad (37)$$

$$s.t. \int_0^1 (s(X)-I)\pi(h|X)f(X)dX \geq 0, \quad and \quad \pi(h|X)\in[0,1].$$

In Eq. (37), the entrepreneur maximizes his expected payoff given the participation constraint for the investors. Note that $\mathbb{E}[s(X)|X\geq I-\varepsilon]\geq I$. Therefore, $s(X)$ exceeds I with a positive probability. Furthermore, the set of measurable functions satisfying the constraint in Eq. (37) is a closed and bounded subset of $\mathcal{L}^1$. As a result, Eq. (37) has a solution.

The participation constraint in Eq. (37) could be either binding or nonbinding. When it is nonbinding, the optimal experiment sets $\pi^*(h|X)=1$ for all values of X for which the value in the bracket is nonnegative. It corresponds to the set $\{X\geq \hat{X}(\mu)\}$. When the constraint is binding, we apply Lemma A1 with $N=1$. Let $\hat{\lambda}$ be the corresponding multiplier. Then the optimal experiment $\pi^*(h|X)$ solves

$$\max_{\pi(h|X)} \int_0^1 [\varepsilon + X - I + (\hat{\lambda}-\mu)(s(X)-I)]\pi(h|X)f(X)dX. \quad (38)$$

The term in the bracket is strictly increasing in X because

$$\frac{d_+}{d_+X}[\varepsilon + X - I + (\hat{\lambda}-\mu)(s(X)-I)] = 1 + (\hat{\lambda}-\mu)\frac{d_+}{d_+X}s(X) > 1 - \mu\frac{d_+}{d_+X}s(X) \geq 1-\mu \geq 0, \quad (39)$$

where $\frac{d_+}{d_+X}$ denotes the right derivative. Therefore, the optimal experiment has a threshold scheme, where the threshold $\bar{X}$ satisfies $\int_{\bar{X}}^1 (s(X)-I)f(X)dX = 0$. Since the constraint in this case is binding, $\hat{X}(\mu)\leq\bar{X}$, whereas the opposite holds in the first case. The entrepreneur thus always follows a threshold strategy where the threshold is given by $\max\{\bar{X}, \hat{X}(\mu)\}$. The uniqueness of the payoffs follows directly from Lemma A1(a).

Uniqueness and mixed strategies. So far, we have assumed that the insider follows a pure strategy. As follows, we prove that even if the insider can randomize between continuation and termination following some interim signals, the same essentially unique equilibrium ensues.

To see this, we show that there exists no mixed strategy Nash equilibrium in which the insider financier terminates the project with a positive probability when she is indifferent between continuation and termination. Suppose the contrary that the entrepreneur uses experiment $(\mathcal{Z}', \pi')$ and the insider uses the investment function $i'(.): \mathcal{Z}'\to[0,1]$. If the insider randomizes following some signal realization $z'\in\mathcal{Z}'$, i.e., $i'(z')\in(0,1)$, it implies that the insider should be indifferent between continuation and termination after observing z', (i.e., $\mathbb{E}[s(X)-I|z']=0$). If z' realizes with a positive probability, then there exists $X_{z'}\in(0,1)$ such that $P(X\geq X_{z'}|z')>i'(z')$. Then, consider an alternative experiment $(\mathcal{Z}'', \pi'')$ that splits signal z' to signals z'_h and z'_l, where $\mathcal{Z}''=(\mathcal{Z}'\setminus\{z'\})\cup\{z'_h, z'_l\}$ and $\pi''(z'_h|X)=\pi'(z'|X)\mathbb{I}_{\{X\geq X_{z'}\}}$ and $\pi''(z'_l|X)=\pi'(z'|X)\mathbb{I}_{\{X<X_{z'}\}}$ and $\pi''(z''|X)=\pi'(z''|X)$ for all $z''\in\mathcal{Z}'\setminus\{z'\}$. We have

$$\mathbb{E}[s(X)-I|z'_h] > \mathbb{E}[s(X)-I|z'] = 0 > \mathbb{E}[s(X)-I|z'_l],$$
$$\text{and } U^E(\mathcal{Z}'', \pi'') - U^E(\mathcal{Z}', \pi') = P(z')[P(X\geq X_{z'}|z')\mathbb{E} \times [\varepsilon + X - s(X)|z', X\geq X_{z'}] - i'(z')\mathbb{E}[\varepsilon + X - s(X)|z']] > 0, \quad (40)$$

because $P(X\geq X_{z'}|z')>i'(z')$ by construction and $\mathbb{E}[\varepsilon+X-s(X)|z', X\geq X_{z'}]\geq\mathbb{E}[\varepsilon+X-s(X)|z']$ due to the monotonicity of $X-s(X)$. It contradicts the optimality of $(\mathcal{Z}', \pi')$. As such, any outcome that involves randomization by the insider cannot emerge as an equilibrium outcome.

A3. Proof of Corollary 1

Remember that the corollary does not assert that relationship financing is always feasible in $[\mu^l, \mu^h]$. The key message is that there are regions in which relationship financing breaks down.

The insider's equilibrium interim payoff is as follows:

$$U^I(\{h,l\}, \pi^*(\mu); \mu) = \mu\mathbb{E}[(s(X)-I)\mathbb{I}_{\{X\geq\max\{\bar{X}, \hat{X}(\mu)\}\}}], \quad (41)$$

where $\pi^*(\mu)$ denotes the optimal experiment for μ. It is easy to see that the insider's payoff is zero for $\mu=0$ and $\mu=1$. Given that U^I is continuous in μ, the insider's expected payoff is less than K for a neighborhood around $\mu=0$ and $\mu=1$. The corollary follows.

A4. Proof of Proposition 2

A design implements the socially optimal outcome when the investment takes place iff $X\geq I-\varepsilon$. We introduce a security that maximizes the entrepreneur's expected payoff and implements the socially optimal outcome. Then we characterize the set of optimal designs that achieves the first-best regardless of the realization of ε.

Social optimality of optimal designs. Note that the social surplus from the relationship financing for a given ε is bounded by

$$U_{FB}(\varepsilon) = \mathbb{E}[(\varepsilon + X - I)\mathbb{I}_{\{\varepsilon+X-I\geq 0\}}] - K. \tag{42}$$

We show that this bound is achievable for a contract that satisfies the constraints in conditions (18)–(21). Note that condition (19) implies that $M(X; s_I, s_O, \lambda, \varepsilon) = \varepsilon + X - I$ for all $X \in [\lambda I, I]$, where $M(\cdot)$ is defined in Eq. (14). Since $M(\cdot)$ is increasing in X and $M(I-\varepsilon; s_I, s_O, \lambda, \varepsilon) = 0$, the entrepreneur sends a high signal for $X \geq I - \varepsilon$, provided the security can cover the investment cost for the insiders and outsiders. Condition (20) ensures that is the case for the insider. Furthermore, the condition ensures the entrepreneur can raise $(1-\lambda)I$ from the outsiders by issuing $s_O(\cdot)$ since

$$\begin{aligned}
&\int_{I-\varepsilon}^{1} (s_O(X) - (1-\lambda)I) f(X)dX \\
&= \int_{I-\varepsilon}^{1} (X - s_I(X) - (1-\lambda)I) f(X)dX \\
&= \int_{I-\varepsilon}^{1} (X - I) f(X)dX - \int_{I-\varepsilon}^{1} (s_I(X) - \lambda I) f(X)dX \\
&= \int_{I-\varepsilon}^{1} (X - I) f(X)dX - K = \mathbb{E}[(X-I)\mathbb{I}_{\{X\geq I-\varepsilon\}}] - K \geq 0 \\
&\Rightarrow p^O(\varepsilon) - (1-\lambda)I = \mathbb{E}[s_O(X) - (1-\lambda)I | X \geq I - \varepsilon] \geq 0 \\
&\quad \forall \varepsilon \in (0, \bar{\varepsilon}),
\end{aligned} \tag{43}$$

where we used the conditions (19) and (20) in the third equation. As a result, the entrepreneur receives expected payoff $U_{FB}(\varepsilon)$ for all $\varepsilon \in (0, \bar{\varepsilon})$ and the socially optimal outcome is implemented. We have proven that all optimal designs implement the socially optimal outcome.

The set of optimal designs. We now argue that the set of contracts specified in Eqs. (18)–(21) are the only optimal designs. For a contract to implement the socially efficient outcome for all values of ε, we need to have $M(I-\varepsilon; s_I, s_O, \lambda, \varepsilon) = 0$ for all $\varepsilon \in (0, \bar{\varepsilon})$. Therefore, we should have $s_I(I-\varepsilon) = \lambda I$ for all $\varepsilon \in (0, \bar{\varepsilon})$. It proves the necessity of condition (19) for $X > I - \bar{\varepsilon}$. Furthermore, there would be no investment for $X < I - \bar{\varepsilon}$, as it would be inefficient for any value of ε. It implies the contingent transfers for these states are irrelevant.

According to condition (19), we need to have $\lambda I = s_I(I-\varepsilon) \leq I - \varepsilon$ for all $\varepsilon \in (0, \bar{\varepsilon})$, which implies that Eq. (18) needs to hold as well. Condition (20) ensures that the insider breaks even over the course of the relationship. Finally, the inequality in Eq. (43) becomes equality as ε goes to $\bar{\varepsilon}$. Therefore, the limited liability condition has to bind to implement the socially optimal investment decision when the entrepreneur's private benefit is large.

A5. Proof of Lemma 1

We first prove a useful lemma:

Lemma 2. The optimal (Z, π) is at least as informative as (q, ω_q), in the Blackwell sense. In other words, the outsiders perfectly infer $y \in \mathcal{Y}$ by observing $z \in \mathcal{Z}$ from the endogenous experimentation.

Proof. We show that the entrepreneur designs the experiment in a way that the realized signal z fully reveals the insider's signal y. In other words, for a given signal $z \in \mathcal{Z}$ in the optimal experiment, there exists signal $y \in \mathcal{Y}$ such that $P(y|z) = \frac{\int_0^1 \pi(z,y|X) f(X)dX}{\sum_{y' \in \mathcal{Y}} \int_0^1 \pi(z,y'|X) f(X)dX} = 1$.

Consider experiment $(\mathcal{Z}, \pi)$ and signal $z \in \mathcal{Z}$. Suppose there are $l \geq 2$ distinct signals $\tilde{\mathcal{Y}}(z) = \{y_1, y_2, \ldots, y_l\} \subset \mathcal{Y}$ such that $P(y_i|z) > 0$ for all $1 \leq i \leq l$. We show that the entrepreneur can increase his expected payoff by splitting signal z into signals $z_1, z_2, \ldots, z_l$, where $\pi(z_i, y_j|X) = \pi(z, y_j|X)\mathbb{I}_{i=j}$.

It should be apparent that the insider either chooses $\lambda = 1$ or $\lambda = 0$ because her expected payoff is linear in her amount of investment. Suppose the insider makes the investment for a subset $\tilde{\mathcal{Y}}^+(z) \subsetneq \tilde{\mathcal{Y}}(z)$, following signal z. When signal z is public and $\lambda = 0$, the outsiders offer $p^O = \mathbb{E}[s(X)|z, y \in \tilde{\mathcal{Y}}(z) \setminus \tilde{\mathcal{Y}}^+(z)]$ if it exceeds the cost of investment I; otherwise they do not make any offer. We argue that $p^O < I$ (i.e., that is the outsiders never make any offer when the insider has a strictly more informative signal).

Consider the contrary and suppose $p^O = \mathbb{E}[s(X)|z, y \in \tilde{\mathcal{Y}}(z) \setminus \tilde{\mathcal{Y}}^+(z)] \geq I$. Then there should exist $\tilde{y} \in \tilde{\mathcal{Y}}(z) \setminus \tilde{\mathcal{Y}}^+(z)$ such that $\mathbb{E}[s(X)|z, \tilde{y}] \geq I$. It implies that the insider should also continue after $(z, \tilde{y})$, or equivalently, $\tilde{y}$ needs to be in $\tilde{\mathcal{Y}}^+(z)$ as well, which is a contradiction. Therefore, the insider never pays more than I following such signal z, even if z is publicly observed.

In contrast, if the entrepreneur splits the signals into $z_1, z_2, \ldots, z_l$ as mentioned above, he gets strictly more than I for the realization of z_i and y_i that $\mathbb{E}[s(X)|z_i, y_i] > I$ when z_i is publicly observed. Therefore, the entrepreneur would indeed be better off by the split. □

Lemma 2 essentially says that the entrepreneur optimally conveys the insider's private signal to the outsiders. This does not matter when z is only observed by the insider (with probability μ). But when z is public (with probability $1-\mu$), the entrepreneur prefers to level the playing field by informing the outsiders of y and by eliminating the insider's informational advantage. A priori, one might think this could hurt the entrepreneur's payoff because a negative y signal may decrease the probability of continued financing from outsiders. But obfuscating the signal is not helpful here because the insider's termination action upon seeing a negative y already conveys the information to the outsiders.[25]

As such, the entrepreneur essentially faces m different experiment design problems, each specified by Eq. (7) with the priors $f(X|y_i)$. Proposition 1 then leads us to the optimal experiments under investor sophistication. The only exceptions are the cases in which either $P(s(X) \geq I|y_i) = 0$ or $\mathbb{E}[X|y_i] > 0$, where $\bar{X}(y_i)$ does not exist. In the first case, there would be no investment by the insider, and consequently the outsiders (according to Lemma 2), regardless of the entrepreneur's choice of signals. In the second case,

[25] Note the lemma relies on the entrepreneur's ability in designing an experiment that nests the insider's experiment. This is not crucial for the results to follow, as we demonstrate in Online Appendix OA5.

the entrepreneur optimally induces investment only when $X \geq \hat{X}(\mu)$.

A6. Proof of Proposition 3

Proof of part (a)

As follows, we show that the insider financier earns a higher expected payoff from a more informative experiment. Consider two experiments $(\mathcal{Y}, \omega_q)$ and $(\mathcal{Y}, \omega_{q'})$ with $q' > q$. According to Blackwell (1953), there exists an $m \times m$ Markovian matrix T such that $f_q(X|y_i) = \sum_{j=1}^{m} T_{ij} f_{q'}(X|y_j)$. Moreover, we can write the insider's expected payoff from experiment $(\mathcal{Y}, \omega_q)$ as

$$U^I(\mu; q) = \mu \sum_{y \in \mathcal{Y}} P_q(y_i) \mathbb{E}\big[(s(X) - I)\mathbb{I}_{\{X \geq \max\{\hat{X}(\mu), \bar{X}(y)\}\}}\big]. \tag{44}$$

According to the definition of $\bar{X}(y)$ introduced in Lemma 1, $\bar{X}(y) > 0$ implies that $\mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq \bar{X}\}}] = 0$. We can thus rewrite Eq. (41) as

$$U^I(\mu; q) = \mu \sum_{i=1}^{m} P_q(y_i) \max\{\int_{\hat{X}(\mu)}^{1} (s(X) - I) f_q(X|y_i) dX, 0\}. \tag{45}$$

Substituting $f_q(X|y_i)$ by $\sum_{j=1}^{m} T_{ij} f_{q'}(X|y_j)$, we have

$$\begin{aligned}
U^I(\mu; q) &= \mu \sum_{i=1}^{m} P_q(y_i) \max\left\{\int_{\hat{X}(\mu)}^{1} (s(X) - I) \sum_{j=1}^{m} T_{ij} f_{q'}(X|y_j) dX, 0\right\} \\
&\leq \mu \sum_{i=1}^{m} P_q(y_i) \sum_{j=1}^{m} T_{ij} \max\left\{\int_{\hat{X}(\mu)}^{1} (s(X) - I) f_{q'}(X|y_j) dX, 0\right\} \\
&= \mu \sum_{i=1}^{m} P_{q'}(y_i) \max\left\{\int_{\hat{X}(\mu)}^{1} (s(X) - I) f_{q'}(X|y_j) dX, 0\right\} \\
&= U^I(\mu; q'),
\end{aligned}$$

where the last inequality follows from the identity $P_{q'}(y_j) = \sum_{i=1}^{m} T_{ij} P_q(y_i)$.

Proof of part (b)

Suppose the contrary that there exists an insider's experiment $(\mathcal{Y}, \omega_q)$ that leads to the socially optimal investment decisions. To implement the socially optimal outcome, the threshold for all m signals should be $I - \varepsilon$. We thus need to have $\max\{\hat{X}(\mu), \bar{X}(y)\} = I - \varepsilon$ for all $y \in \mathcal{Y}$. Since $\hat{X}(\mu) < I - \varepsilon$ for all $\mu > 0$, we need to have $\bar{X}(y) = I - \varepsilon$ for all signals in $\mathcal{Y}$. Therefore, by definition of $\bar{X}(y)$, we have $\mathbb{E}[s(X) - I|y_i, X \geq I - \varepsilon] = 0$ for all $1 \leq i \leq m$. Therefore, even though the optimal experiment is socially efficient, the insider receives zero interim expected payoff, failing to recover the initial cost K. Then the insider would not start the relationship financing in the first place, contradicting the outcome being socially optimal.

A.7. Proof of Proposition 4

The derivative of Eq. (44) with respect to μ (when it exists) is

$$\begin{aligned}
&\frac{d}{d\mu} U^I(\mu; q) \\
&= \sum_{y \in \mathcal{Y}} P_q(y) \mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq \max\{\hat{X}(\mu), \bar{X}_q(y)\}\}}] \\
&\quad + \mu \sum_{y \in \mathcal{Y}} P_q(y)(s(\hat{X}(\mu)) - I) f(\hat{X}(\mu)|y) \mathbb{I}_{\{\hat{X}(\mu) > \bar{X}_q(y)\}}.
\end{aligned} \tag{46}$$

To derive the relation between the insider's expected payoff and μ, fix q and consider two cases:

1. Suppose $\mu \geq 1 - \frac{\varepsilon}{I}$, which implies $\hat{X}(\mu) = 0$. Then Eq. (41) implies that $U^I(\cdot\,; q)$ is weakly increasing in μ for $\mu \in [1 - \frac{\varepsilon}{I}, 1]$.
2. Suppose $\mu < 1 - \frac{\varepsilon}{I}$; then $\hat{X}(\mu) > 0$. In this range of values of μ, if $\bar{X}_q(y) < \hat{X}(\mu)$ for some $y \in \mathcal{Y}$, then the first term in the right-hand side of Eq. (46) is positive and the second term is negative. For small enough values of μ, the derivative is strictly positive since the first term dominates the second term. Moreover, the derivative is weakly decreasing since both of the terms are decreasing in μ. It implies the insider's expected payoff is concave in μ for $\mu \in [0, 1 - \frac{\varepsilon}{I}]$.

Denote $\bar{\mu} \in [0, 1 - \frac{\varepsilon}{I}]$ the maximizer of $U^I(\cdot\,; q)$. If $\bar{\mu} < 1 - \frac{\varepsilon}{I}$, then the insider's expected payoff is U-shaped in μ for $\mu \in [\bar{\mu}, 1]$, which completes the proof.

A8. Proof of Proposition 5

For every insider's experiment $(\mathcal{Y}, \omega_q)$, conditions (19)–(21) characterize the set of optimal long-term contracts. In particular, we show that under these conditions, the entrepreneur optimally designs a binary experiment that sends a high signal if $X \geq I - \varepsilon$, which induces investment. First, suppose the entrepreneur chooses this experiment. Condition (19) implies the insider always invests if she learns that $X \geq I - \varepsilon$. Conditions (20) and (21) together imply the outsiders also invest if and only if the entrepreneur's experiment sends a high signal. The reason is that the insider's action is binary and it only reveals the signal of the entrepreneur's experiment. Therefore, the project is invested if and only if $X \geq I - \varepsilon$. Moreover, by an argument similar to the proof of Proposition 2, it is the optimal experiment for the entrepreneur, and these contracts yield the entrepreneur the whole social surplus. Now to show that the optimal design has to satisfy conditions (19)–(21), we can use the argument almost verbatim in the proof of Proposition 2.

A9. Proof of Corollary 2

It is easy to show that Lemma 2 still holds: the entrepreneur chooses an experiment strictly more informative than $(\mathcal{Y}, \omega_q)$. It means the entrepreneur still solves m independent information design problem for every signal in $\mathcal{Y}$ to determine the additional information to reveal.

This independence implies that the entrepreneur does not choose a more informative experiment for signals in $\mathcal{Y}^{nb}$, compared to the benchmark case without setting milestone. Moreover, the insider's action following signals in $\mathcal{Y}^{b}$ is weakly dominated by that without the commitment because in the latter she can optimally respond to the additional information the entrepreneur provides. Therefore, the insider does not gain from setting milestones.

A10. Proof of Corollary 3

Similar to the earlier discussions, one can show the entrepreneur chooses a binary experiment with a threshold structure for both cases. Denote the threshold values for securities $s_1(X)$ and $s_2(X)$ by $\bar{X}_1^c$ and $\bar{X}_2^c$, respectively. Eq. (26) directly implies that

$$\mathbb{E}[\{s_1(X)-I\}^-|X\geq\bar{X}_1^c]=-c=\mathbb{E}[\{s_2(X)-I\}^-|X\geq\bar{X}_2^c] \geq\mathbb{E}[\{s_1(X)-I\}^-|X\geq\bar{X}_2^c]\Rightarrow\bar{X}_1^c\geq\bar{X}_2^c. \quad (47)$$

A11. Proof of Proposition 6

Proof for part (a)

Since the insider's payoff is continuous in μ, we only need to prove that the insider receives zero expected payoff for $\mu=0$ and $\mu=1$. For $\mu=0$, the insider has no information rent and clearly gets zero expected payoff. We now discuss the case of $\mu=1$.

Secret manipulation. First suppose the entrepreneur can secretly change the signal realization with probability $\alpha>0$. Similar to Proposition 1, the entrepreneur follows a threshold strategy (i.e., there exists $\bar{X}_\alpha\in[0,1]$ such that the experiment generates a high signal for $X\geq\bar{X}_\alpha$). The high signal induces investment if the investor receives a non-negative expected payoff from the investment following the high signal, which is equivalent to

$$\alpha\int_0^{\bar{X}_\alpha}(s(X)-I)f(X)dX+\int_{\bar{X}_\alpha}^1(s(X)-I)f(X)dX\geq 0. \quad (48)$$

The first term in Eq. (48) shows the probability that the experiment generates a low signal, but the entrepreneur finds the chance to send a high signal. Note that for $\alpha=1$, the inequality does not hold because

$$\mathbb{E}[s(X)-I]<\mathbb{E}[\varepsilon+X-I]<0. \quad (49)$$

Therefore, there exists $\bar{\alpha}\in[0,1]$ above which the investment is not feasible because the entrepreneur's commitment problem to truthful reporting of the signal is sufficiently serious. However, for $\alpha\leq\bar{\alpha}$, the entrepreneur chooses $\bar{X}_\alpha$ such that the inequality (48) binds, which implies the investor becomes indifferent between investment and not investment after receiving the high signal. As a result, the investor receives zero interim rent for all values of $\alpha\in[0,1]$.

Random monitoring. Now consider the case that the insider verifies the signal realization with probability $\beta<1$. This case involves two subcases. First, the investor cannot commit to punishing the entrepreneur for misreporting. In this subcase, there is no signal such as h that always induces investment because otherwise the entrepreneur would optimally always report h, which leads to negative expected payoff for the investor when she does not monitor. As such, the investor only invests when she monitors, which makes the entrepreneur's reporting strategy irrelevant. Hence the entrepreneur would follow the threshold strategy at $\bar{X}$, in which the investor does not any get interim rent.

Second, consider the subcase that the investor commits to punish misreporting. In this case, the entrepreneur might use four different kinds of signals in his experiment: 1) low signals, such as l_0, that never induce investmen; 2) low signals, such as l_1, that only induce investment when the investor does not monitor; 3) high signals, such as h_0, that only induce investment if they are verified; and 4) high signals, such as h_1, that always induce investment. The probability of investment is $(1-\beta)P(l_1)+\beta P(h_0)+P(h_1)$. We next check the incentive constraints for truthful reporting for the entrepreneur.

In particular, we show that there is no equilibrium that the insider invests without monitoring. Consider the contrary. If $\beta<\frac{1}{2}$, then types l_0 and h_0 prefer to report h_1 instead of truthfully reporting because the probability of investment strictly for l_0 and h_0 increases from 0 and β, respectively, to $1-\beta$. Note that in all cases, the insider pays exactly I upon continuation. Therefore, only the probability of investment affects the entrepreneur's payoff at every state X. Because of these, we should have $P(l_0)=P(h_0)=0$, which implies that the investor always invests when she is not monitoring. Thus, she would be better off by not investing at all when she does not monitor, as her investment has a negative NPV conditional on not monitoring. It implies $P(l_1)=P(h_1)=0$ as well and contradicts that the equilibrium involves investment with a positive probability in absence of monitoring.

For $\beta\geq\frac{1}{2}$, $P(l_0)=0$ because he would be strictly better off by reporting h_1. If the investor receives a positive payoff from investment following $h_1\cup l_1$, she should make a loss by investing following h_0 since $P(h_0)+P(l_1)+P(h_1)=1$ and the project has an unconditional negative NPV. Therefore, the investor would be better off by not investing following h_0, even after verifying it, which is a contradiction with its definition. Consequently, the investor never invests without monitoring.

Proof for part (b)

Secret manipulation. We first show that the socially optimal outcome cannot be implemented for $\alpha>0$, and then we show that the optimal contracts feature securities with a flat region for the insider (i.e., condition (19) holds).

Note that once the entrepreneur raises K, he always wants to continue the project since he receives a strictly positive payoff from continuation. Consequently, the entrepreneur misreports the bad signals whenever possible, which leads to inefficient continuation. Therefore, the socially optimal outcome is not implementable for $\alpha>0$.

Now we solve for the optimal contract. If the project is invested with a positive probability, then the expected payoff of the entrepreneur from the contract $\{s_I(.), s_O(.), \lambda\}$ is

$$U_\alpha^E(s_I(\cdot),s_O(\cdot),\lambda,\varepsilon) =\mathbb{E}[M(X;s_I,s_O,\lambda,\varepsilon)(\alpha+(1-\alpha)\mathcal{I}(X;\varepsilon))], \quad (50)$$

where $\mathcal{I}(\cdot)$ is the investment function for the case that the entrepreneur cannot secretly manipulate the signal. If

$\mathbb{E}[\max\{X - I + \varepsilon, 0\}] > K$, then with an argument similar to the proof of Proposition 2, the following set of convertible securities are optimal and independent of ε, for small enough values of α:

$$\lambda I \le I - \bar{\varepsilon}, \quad s_I(X) = \min\{\lambda I, X\} \forall\ X < I,$$
$$\mathbb{E}[(s_I(X) - \lambda I)(\alpha + (1-\alpha)\mathbb{I}_{\{X \ge I - \varepsilon\}})] = K, \tag{51}$$

$$\mathbb{E}[(s_O(X) - (1-\lambda)I)(\alpha + (1-\alpha)\mathbb{I}_{\{X \ge I - \varepsilon\}})] = 0,$$
$$0 \le s_O(X) \le X - s_I(X) \quad \forall\ X \in [0, 1]. \tag{52}$$

Note that the design might not be robust to the insider's experiment $(\mathcal{Y}, \omega_q)$ because the insider's payoff becomes sensitive to the downside realization of the final cash flow. Moreover, for big enough values of α, no security can satisfy condition (51). Therefore, relationship financing is infeasible for such big values of α.

Random monitoring. Consider the case that the investor cannot credibly threaten the entrepreneur to terminate the project when he misreports. The argument for the other case is similar. As discussed earlier, in equilibrium, the entrepreneur always reports a high signal, and the investor invests if and only if she verifies the signal is truthfully reported. Therefore, the socially optimal outcome cannot be implemented when $\beta < 1$. Moreover, the following convertible securities are optimal.

$$\lambda I \le I - \bar{\varepsilon}, \quad s_I(X) = \min\{\lambda I, X\}$$
$$\forall\ X < I, \quad \beta\mathbb{E}[(s_I(X) - \lambda I)\mathbb{I}_{\{X \ge I - \varepsilon\}}] = K,$$
$$\mathbb{E}[(s_O(X) - (1-\lambda)I)(\alpha + (1-\alpha)\mathbb{I}_{\{X \ge I - \varepsilon\}})] = 0,$$
$$0 \le s_O(X) \le X - s_I(X) \quad \forall\ X \in [0, 1]. \tag{53}$$

Clearly, these securities are implementable for large enough values of β. For the case of credible punishments, an optimal design might not exist because the optimal experimentation involves three signals for large values of ε, while it involves two signals for the smaller values. However, for smaller values of ε, the convertible securities specified above are optimal and the equilibrium outcomes are similar.

References

Aghion, P., Dewatripont, M., Rey, P., 1994. Renegotiation design with unverifiable information. Econom.: J. Econom. Soc. 62, 257–282.
Anand, B.N., Galetovic, A., 2006. Relationships, competition and the structure of investment banking markets. J. Ind. Econ. 54 (2), 151–199.
Bergemann, D., Morris, S., 2016. Information design, Bayesian persuasion, and Bayes correlated equilibrium. Am. Econ. Rev. 106 (5), 586–591.
Berger, A.N., Udell, G.F., 1995. Relationship lending and lines of credit in small firm finance. J. Bus. 68, 351–381.
Black, S.E., Strahan, P.E., 2002. Entrepreneurship and bank credit availability. J. Financ. 57 (6), 2807–2833.
Blackwell, D., 1953. Equivalent comparisons of experiments. Ann. Math. Stat. 24, 265–272.
Boot, A.W., Thakor, A.V., 2000. Can relationship banking survive competition? J. Financ. 55 (2), 679–713.
Bouvard, M., Chaigneau, P., Motta, A.D., 2015. Transparency in the financial system: rollover risk and crises. J. Financ. 70 (4), 1805–1837.
Brennan, M.J., Schwartz, E.S., 1988. The case for convertibles. J. Appl. Corp. Financ. 1 (2), 55–64.
Chang, B., Szydlowski, M., 2020. The market for conflicted advice. J. Financ. 75 (2), 867–903.
Cong, L.W., 2017. Auctions of Real Options. Chicago Booth School of Business. Unpublished working paper
Cong, L.W., Grenadier, S.R., Hu, Y., 2020. Dynamic interventions and informational linkages. J. Financ. Econ. 135, 1–15.
Cong, L.W., Xiao, Y., 2018. Information Cascades and Threshold Implementation. The Chinese University of Hong Kong and University of Chicago. Unpublished Working Paper
Cornelli, F., Yosha, O., 2003. Stage financing and the role of convertible securities. Rev. Econ. Stud. 70 (1), 1–32.
Degryse, H., Ongena, S., 2007. The impact of competition on bank orientation. J. Financ. Intermed. 16 (3), 399–424.
Dell'Ariccia, G., Marquez, R., 2004. Information and bank credit allocation. J. Financ. Econ. 72 (1), 185–214.
DeMarzo, P., Duffie, D., 1999. A liquidity-based model of security design. Econometrica 67 (1), 65–99.
DeMarzo, P.M., Kremer, I., Skrzypacz, A., 2005. Bidding with securities: auctions and security design. Am. Econ. Rev. 95 (4), 936–959.
Diamond, D.W., 1984. Financial intermediation and delegated monitoring. Rev. Econ. Stud. 51 (3), 393–414.
Diamond, D.W., 1991. Monitoring and reputation: the choice between bank loans and directly placed debt. J. Political Econ. 99 (4), 689–721.
Dinc, I.S., 2000. Bank reputation, bank commitment, and the effects of competition in credit markets. Rev. Financ. Stud. 13 (3), 781–812.
Dworczak, P., Martini, G., 2019. The simple economics of optimal persuasion. J. Political Econ. 127 (5), 1993–2048.
Elsas, R., 2005. Empirical determinants of relationship lending. J. Financ. Intermed. 14 (1), 32–57.
Ely, J.C., 2017. Beeps. Am. Econ. Rev. 107 (1), 31–53.
Ewens, M., Rhodes-Kropf, M., Strebulaev, I.A., 2016. Insider Financing and Venture Capital Returns. California Institute of Technology, Harvard Business School, and Stanford University. Unpublished working paper
Fama, E.F., 1985. What's different about banks? J. Monet. Econ. 15 (1), 29–39.
Fluck, Z., Garrison, K., Myers, S.C., 2006. Venture Capital Contracting: Staged Financing and Syndication of Later-stage Investments. National Bureau of Economic Research. Unpublished working paper
Gentzkow, M., Kamenica, E., 2016. A Rothschild-Stiglitz approach to Bayesian persuasion. Am. Econ. Rev. 106 (5), 597–601.
Goldstein, I., Leitner, Y., 2018. Stress tests and information disclosure. J. Econ. Theory 177, 34–69.
Gompers, P. A., Gornall, W., Kaplan, S. N., Strebulaev, I. A., 2020. How do Venture Capitalists Make Decisions?
Guo, Y., Shmaya, E., 2019. The interval structure of optimal disclosure. Econometrica 87 (2), 653–675.
Hart, O., Moore, J., 1988. Incomplete contracts and renegotiation. Econom.: J. Econom. Soc. 56, 755–785.
Hart, O., Moore, J., 1995. Debt and seniority: an analysis of the role of hard claims in constraining management. Am. Econ. Rev. 85 (3), 567–585.
Hellmann, T., 2006. IPOs, acquisitions, and the use of convertible securities in venture capital. J. Financ. Econ. 81 (3), 649–679.
Holmstrom, B., 1979. Moral hazard and observability. Bell J. Econ. 10, 74–91.
Inderst, R., Vladimirov, V., 2019. Growth firms and relationship finance: a capital structure perspective. Manag. Sci. 65, 4951–5448.
Innes, R.D., 1990. Limited liability and incentive contracting with ex-ante action choices. J. Econ. Theory 52 (1), 45–67.
Inostroza, N., Pavan, A., 2018. Persuasion in Global Games with Application to Stress Testing. Northwestern University. Unpublished working paper
Ito, K., 2016. Functional Analysis and Optimization. North Carolina State University. Unpublished working paper.
Kamenica, E., Gentzkow, M., 2011. Bayesian persuasion. Am. Econ. Rev. 101 (6), 2590–2615.
Kaplan, S.N., Strömberg, P., 2003. Financial contracting theory meets the real world: an empirical analysis of venture capital contracts. Rev. Econ. Stud. 70 (2), 281–315.
Kaplan, S.N., Strömberg, P.E., 2004. Characteristics, contracts, and actions: evidence from venture capitalist analyses. J. Financ. 59 (5), 2177–2210.
Kerr, W.R., Nanda, R., Rhodes-Kropf, M., 2014. Entrepreneurship as experimentation. J. Econ. Perspect. 28 (3), 25–48.
Kolotilin, A., 2018. Optimal information disclosure: a linear programming approach. Theoret. Econ. 13 (2), 607–635.
Kolotilin, A., Mylovanov, T., Zapechelnyuk, A., Li, M., 2017. Persuasion of a privately informed receiver. Econometrica 85 (6), 1949–1964.
Megginson, W.L., Weiss, K.A., 1991. Venture capitalist certification in initial public offerings. J. Financ. 46 (3), 879–903.
Nachman, D.C., Noe, T.H., 1994. Optimal design of securities under asymmetric information. Rev. Financ. Stud. 7 (1), 1–44.
Nöldeke, G., Schmidt, K.M., 1998. Sequential investments and options to own. RAND J. Econ. 29, 633–653.

Orlov, D., Zryumov, P., Skrzypacz, A., 2019. Design of Macro-prudential Stress Tests. Stanford University. Unpublished working paper
Petersen, M.A., Rajan, R.G., 1994. The benefits of lending relationships: evidence from small business data. J. Financ. 49 (1), 3–37.
Petersen, M.A., Rajan, R.G., 1995. The effect of credit market competition on lending relationships. Q. J. Econ. 110 (2), 407–443.
Petersen, M.A., Rajan, R.G., 2002. Does distance still matter? The information revolution in small business lending. J. Financ. 57 (6), 2533–2570.
Rajan, R.G., 1992. Insiders and outsiders: the choice between informed and arm's-length debt. J. Financ. 47 (4), 1367–1400.
Ramakrishnan, R.T., Thakor, A.V., 1984. Information reliability and a theory of financial intermediation. Rev. Econ. Stud. 51 (3), 415–432.
Santos, J.A., Winton, A., 2008. Bank loans, bonds, and information monopolies across the business cycle. J. Financ. 63 (3), 1315–1359.
Schenone, C., 2010. Lending relationships and information rents: do banks exploit their information advantages? Rev. Financ. Stud. 23 (3), 1149–1199.
Sharpe, S.A., 1990. Asymmetric information, bank lending, and implicit contracts: a stylized model of customer relationships. J. Financ. 45 (4), 1069–1087.
Stein, J.C., 1992. Convertible bonds as backdoor equity financing. J. Financ. Econ. 32 (1), 3–21.
Szydlowski, M., 2020. Optimal financing and disclosure. Manag. Sci. forthcoming (online on May 14).
Trigilia, G., 2019. Optimal Leverage, Transparency, and the Decision to go Public. University of Warwick. Unpublished working paper
Von Thadden, E.-L., 1995. Long-term contracts, short-term investment and monitoring. Rev. Econ. Stud. 62 (4), 557–575.
Yafeh, Y., Yosha, O., 2001. Industrial organization of financial systems and strategic use of relationship banking. Rev. Financ. 5 (1–2), 63–78.
Yang, M., Zeng, Y., 2018. Financing entrepreneurial production: security design with flexible information acquisition. Rev. Financ. Stud. 32 (3), 819–863.